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DEBATE PARA LA GENERACIÓN DE POLÍTICAS

ÍNDICE

INTRODUCCIÓN

Santiago Castro Gómez
Presidente Asobancaria

Los esfuerzos por vincular a la academia en el quehacer del sector bancario y financiero colombiano se han materializado, por segundo año consecutivo, en nuestro Call for Papers Asobancaria. Este concurso ha tenido como fin divulgar e incentivar investigaciones de alto rigor que también tengan una aplicación práctica para nuestras actividades, y busquen dar respuesta, en un sentido amplio, a los desafíos que surgen en nuestra actividad.

El país posee programas académicos de economía y finanzas de alta calidad, con un cuerpo docente altamente capacitado, destacado no solo al interior del país, sino también en el ambiente internacional, quienes cultivan día a día el espíritu investigador para hacer de nuestros estudiantes profesionales íntegros y apasionados. No obstante, desde el gremio hemos notado que la participación relativa de la producción de documentos académicos con temáticas asociadas a economía, econometría y finanzas en los últimos años se ha reducido.

Es por ello que hemos querido fomentar y estrechar los vínculos entre el sector y la academia. Desde el 2017 se hizo realidad nuestro proyecto Call For Papers y el premio a la Mejor Tesis de Maestría en Economía y Finanzas del país, con el propósito de brindarle un espacio adicional a la investigación y avanzar hacia la consecución de un mercado de capitales cada vez más competitivo. En la segunda edición de los premios, tuvimos la oportunidad de escuchar las ponencias de los ganadores y sus consideraciones acerca de temas relacionados con lasineficiencias y su relación con el desempeño bancario, la liquidez durante situaciones de estrés financiero y los retos de los fondos de inversión colectiva en Colombia.

Me complace presentar las memorias de las investigaciones ganadoras del Call For Papers para la versión 2018. Para las decenas de participantes y al selecto grupo de jurados, quiero extenderles mi agradecimiento por confiar en esta iniciativa. Felicitamos, desde luego, a los ganadores, para quienes estas memorias resultan ser el mejor honor a su trabajo.

Finalmente, invitamos a toda la comunidad académica y profesional a participar en la edición de 2019, la cual aceptará aplicaciones hasta el 109 de mayo. El reconocimiento y la presentación de los trabajos ganadores se llevará a cabo en el 31° Simposio de Mercado de Capitales, evento que tendrá lugar los días 22 y 23 de agosto de 2019 en la ciudad de Medellín. Extendemos de ante mano nuestro agradecimiento a los nuevos postulantes. Estamos seguros de que los nuevos trabajos continuarán fortaleciendo nuestro tejido académico.

FIC-DELIZACIÓN: RETOS Y ANÁLISIS CON PROGRAMACIÓN DINÁMICA PARA FONDOS DE RENTA FIJA EN COLOMBIA

Isaza, Esteban* - Restrepo Ochoa, Diana

RESUMEN

Utilizando modelos de índice único y programación dinámica este trabajo evalúa el desempeño de 48 fondos de inversión colectiva en renta fija entre 2010 y 2018. Además, cuantifica para aquellos fondos creadores de valor las medidas de disimilitud cruzadas para su gestión, la de otros fondos en su misma categoría y el benchmark correspondiente. Los resultados soportan la capacidad de los FICs colombianos de renta fija para crear valor y una relación positiva entre la disimilitud en estilos de gestión de los fondos y su desempeño.

01 | Introducción

Recientemente los FICs se han convertido en alternativas atractivas de inversión, ya que permiten a pequeños inversionistas invertir en portafolios diversificados sin necesidad de aportar grandes cantidades de dinero, ni tener experiencia o conocimientos profundos del mercado de capitales ***(Cano y Magán, 2017)***. En los últimos años se ha observado un crecimiento importante en términos del monto invertido en dichos instrumentos, pasando de cerca de 44 billones en el año 2016 a 75 billones en 2018. En particular, los FICs de renta fija nacional en Colombia administran actualmente cerca del 65% del monto total. Por esta razón, una evaluación de su desempeño puede contribuir a un mayor alcance de dichos instrumentos en el mercado y a una mejora en la información disponible para que los inversionistas puedan elegir los fondos que mejor se adapten a sus expectativas de riesgo, rentabilidad y horizonte.

De acuerdo con lo anterior, este artículo evalúa el desempeño de 48 fondos de renta fija en Colombia a través de modelos de índice único, con el fin de determinar si estos generan valor. Adicionalmente, mediante programación dinámica, se calculan medidas de disimilitud sobre series de tiempo de inversiones hipotéticas en estos fondos, para determinar si la creación de valor es resultado de una gestión[1] que se aleja de la norma; es decir, estas series siguen un patrón marcadamente disímil a otros fondos de la misma categoría.

Esta investigación tiene dos contribuciones: en primer lugar, aporta a la literatura de FICs en renta fija con evidencia empírica, que es aún incipiente, considerando la

Chen, Ferson, y Peters (2010) encuentran también que el desempeño promedio de los fondos de renta fija es significativamente negativo luego de considerar costos, pero significativamente positivos sin introducirlos.

importancia económica de estos fondos. En segundo lugar, presenta a la disimilitud en series de tiempo como factor explicativo del desempeño, lo cual contribuye a comparar la gestión de fondos desde sus series de tiempo sin necesidad de recurrir a un meta-análisis de sus fichas técnicas.

El resto del artículo está organizado de la siguiente manera: la sección 2 presenta una revisión de literatura cubriendo estudios previos. La sección 3 describe el contexto y las características generales de los fondos colombianos analizados en este trabajo, en la sección 4 se introducen los datos a utilizar y sus fuentes; luego, la sección 5 propone una evaluación econométrica del desempeño y una aproximación con programación dinámica a los resultados para ambas metodologías se presentan en las secciones 6 y 7. En la sección 8 se establecen las limitaciones del trabajo, así como algunas líneas de investigación futura. Finalmente, la sección 9 concluye.

02 | Literatura

Aunque el acervo de estudios académicos existentes sobre fondos de renta fija es escaso para Colombia, algunos estudios previos en contextos internacionales proveen directrices e hitos con lecciones importantes para el mercado colombiano.

Blake, Elton, y Gruber (1993) usaron modelos de índices lineales y no-lineales en una muestra de fondos de renta fija de NASDAQ entre 1979 y 1988. Los autores encontraron que estos fondos tienen un desempeño inferior al que presentaron en el mismo periodo tiempo algunas estrategias pasivas basadas en índices de renta fija luego de introducir los costos de administración. Añadiendo evidencia de subdesempeño desde una perspectiva internacional global, Detzler (1999) estudia las características de riesgo y retorno para 19 fondos globales, es decir, fondos con posiciones en instrumentos de renta fija de diversos países, entre 1988 y 1995. El autor señala que los fondos con estrategias activas no muestran un desempeño superior, luego de incluir, contra un conjunto de benchmarks entre los que se destaca el índice Salomon Brothers Broad.

Chen, Ferson, y Peters (2010) encuentran también que el desempeño promedio de los fondos de renta fija es significativamente negativo luego de considerar costos, pero significativamente positivos sin introducirlos. Obtienen estos resultados valorando la capacidad de cada fondo de tomar decisiones sincronizadas con el comportamiento del mercado. Los autores definen como

[1] Este trabajo define la gestión en un contexto estocástico; es decir, la gestión de un fondo es la realización específica de una familia de variables aleatorias que agregan activos y valor ordenados en el tiempo. Así, cada realización o gestión es única.

sincronización a la habilidad de usar y procesar información concerniente a las realizaciones futuras de aquellos factores comunes que afectan los retornos de un bono, tales como curvatura, pendiente de la curva y spread de crédito. Sus resultados provienen de una base de datos de 1400 fondos estadounidenses estudiados entre 1962 y 2007.

Usando un modelo multifactorial condicionado para capturar diferencias en plazos de vencimiento y riesgo de default, Ayadi y Kryzanowski (2011) caracterizan el proceso generador de retornos para una muestra de fondos de renta fija en Canadá entre 1984 y 2003. Los resultados sugieren que el desempeño ajustado por riesgo luego de descontar los costos asociados a la administración son negativos y sugieren que ningún fondo exhibe habilidades de gestión extraordinarias.

También hay evidencia de desempeño positivo en estudios académicos, el primer ejemplo a resaltar es el trabajo de Huij y Derwall (2008), quienes investigaron la persistencia del desempeño de 3549 fondos de inversión colectiva de renta fija entre 1990 y 2003 para los Estados Unidos. Su artículo mostró que aquellos fondos que exhibían un desempeño fuerte (débil) en períodos pasados manifestaban una tendencia a mantener este comportamiento en períodos futuros. Recientemente, Moneta (2015), analizando 110 fondos en la base de datos Morningstar, presentan evidencia de que los administradores activos logran un desempeño superior a sus benchmarks antes de costos entre 1997 y 2006.

Diversos trabajos han estudiado los fondos de renta fija y su comportamiento al interior de mercados emergentes. Eling y Faust (2010) muestran que los fondos de cobertura generan un alfa positivo y significativo en mercados emergentes como Latino América, China e India entre 1995 y 2008, mientras que los fondos mutuos no alcanzan a superar el desempeño de los benchmarks tradicionales. Los autores sugieren que estos resultados son explicados por la habilidad de cambiar la ubicación de las inversiones, el fácil acceso a derivados y un estilo de gestión libre por parte de los fondos de cobertura. Huij y Post (2011) encuentran varios fondos colectivos en mercados emergentes capaces de generar retornos y cubrir sus costos de administración, sin embargo, el estudio se realiza sobre fondos mixtos, es decir, con posiciones tanto en renta variable como en renta fija.

La literatura revisada muestra que hay escasez de estudios académicos enfocados exclusivamente al análisis de fondos de renta fija, y la escasez se torna más aguda cuando se restringen los resultados a Colombia.

03 | Contexto Colombiano

Para hacernos una idea de la magnitud de la relación entre fondos de renta fija y renta variable, la figura 1 muestra la evolución histórica de los montos administrados por los 48 fondos que analiza este trabajo, a mayo de 2018. Mientras que estos administraban cerca de 30 billones de pesos, los fondos de renta variable nacional no sobrepasaban los 1,5 billones en el mismo período.

En Colombia, cada fondo pertenece a una única categoría definida por las características de riesgo de los instrumentos que lo componen, estas son definidas por el Sistema de Información de Fondos de Inversión Colectiva Categorizados[2]. El **cuadro 1** desagrega los fondos utilizados por categoría, siendo los FICs de corto plazo los más numerosos con 15 fondos y los FICs de alto rendimiento la categoría más pequeña con solo 4 miembros.

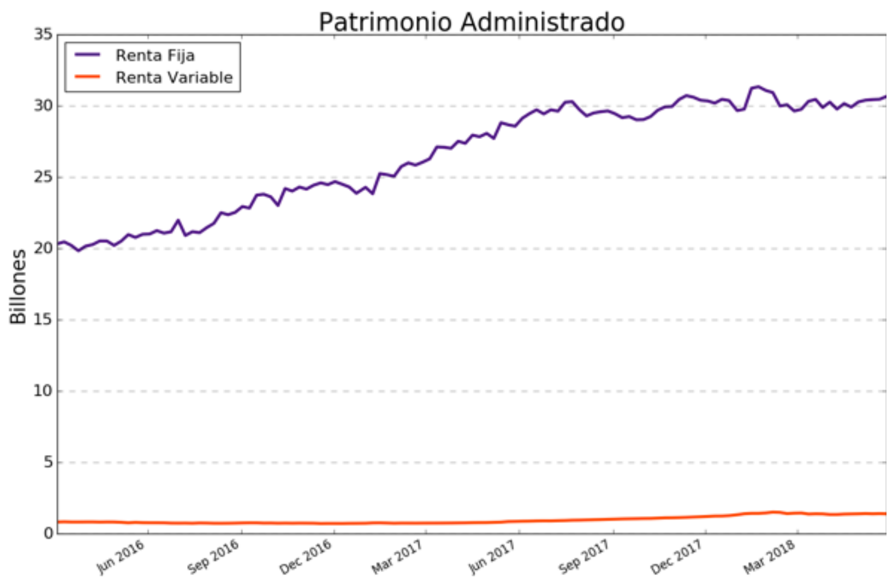


Figura 1: Profundidad de los FICs en Colombia

El cuadro 2 presenta un análisis descriptivo de los fondos analizados por categoría para el periodo de mayo 2017 a mayo 2018. El cuadro 2 revela dos dinámicas interesantes: la primera es un descenso en el patrimonio administrado promedio en los fondos de mediano plazo y largo plazo; segundo, todas las categorías experimentaron rendimientos superiores a 3%.

Si se analizan de forma conjunta el retorno promedio y el crecimiento en inversionistas, el cuadro 2 revela una preferencia por fondos de liquidez y de corto plazo sobre fondos de mediano y largo plazo. Se afirma esto por dos motivos: en primer lugar, los fondos de liquidez y corto plazo experimentaron un alto crecimiento promedio de suscriptores, mientras los fondos de mediano y largo plazo tuvieron una disminución de inversionistas en el mismo periodo. En segundo lugar, en las categorías de liquidez y corto plazo se reportan retornos promedio de 4:54% y 5:1%, respectivamente, mientras que en las categorías de mediano y largo plazo se tienen retornos promedio de 5:15% y 4:49%. Este fenómeno puede deberse a que los inversionistas no perciben mayores rendimientos a medida que aumenta el horizonte de inversión, tal y como propone la hipótesis de preferencia por liquidez (McCulloch, 1975; Fama 1984).

[2] El documento "Metodología Categorización Fondos de Inversión Colectiva 2018", emitido por el Comité de Categorización Interindustria de Asobolsa y Asofiduciarias, establece el marco de funcionamiento de la categorización, define las categorías y describe las reglas específicas para cada una de ellas. https://sificcolombia com/

La figura 2 compara a mayo de 2018 el patrimonio administrado y la profundización de los FICs por categoría. Los FICs analizados administran cerca de 30,4 billones de pesos; en el primer lugar encontramos a los fondos de liquidez administrando cerca de 23 billones (75 %), seguidos por los fondos de corto plazo con 4,6 billones (15 %) bajo su administración. Pasando al conteo de inversionistas, el número de personas naturales o jurídicas que utilizan como instrumento de inversión alguno de los fondos analizados es aproximadamente de 1,3 millones. Los fondos de liquidez concentran cerca de 830 mil inversionistas (63 %), seguidos por los fondos de mediano plazo con aproximadamente 400 mil inversionistas (30 %).

En cuanto a las comisiones, estas no superan el 3% y en promedio son más altas en los fondos de mediano plazo. Clavijo, Vera, y Londoño (2017) consideran que dichas comisiones pueden explicarse como función de la tasa repo, que capturaría el efecto positivo de liquidez-rentabilidad, la valorización del mercado local de capitales y el monto de los activos administrados.

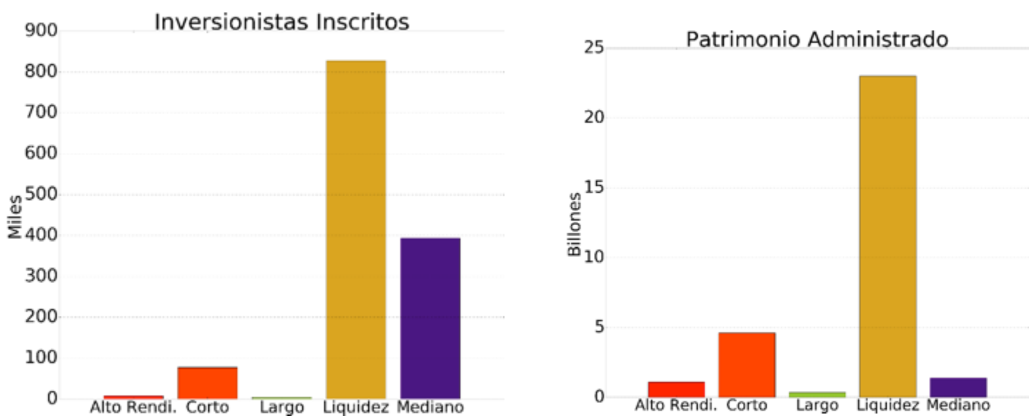
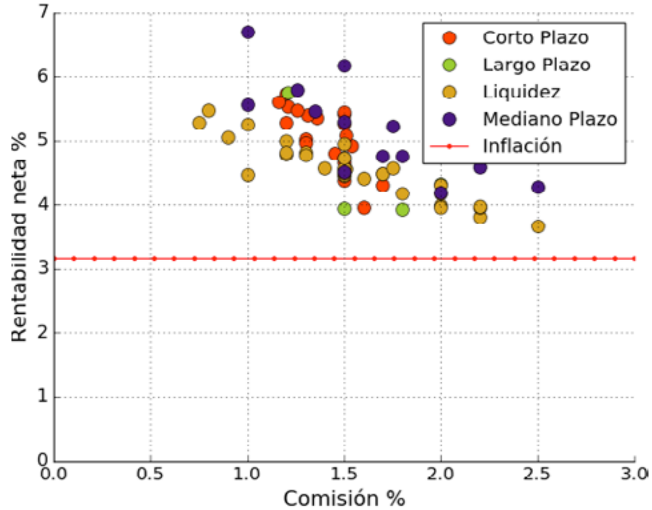


Figura 2: Tamaño y profundización de los FICs

Finalmente, la figura 3 muestra un resultado a resaltar de los FICs de renta fija en Colombia, se evidencia que los fondos son capaces de cumplir con la meta de preservar el poder adquisitivo de los inversionistas, pues logran proporcionar retornos superiores a la inflación en un mismo horizonte de inversión aún después de incorporar la comisión de administración.

Figura 3: Desempeño 12 meses a Mayo 31 de 2018



Liquidez	Min	Media	Max
Inversión inicial (millones)	0.01	1822	21500
Comisión	0.75	1.55	2.5
Rendimiento %Crecimiento	3.66	4.54	5.47
Patrimonio %Crecimiento	-96.3	10.25	339
Inversionistas %	-40.9	15.2	300
Corto Plazo	**Min**	**Media**	**Max**
Inversión inicial (millones)	0.2	171.8	2000
Comisión	1.16	1.43	1.8
Rendimiento %Crecimiento	3.95	5.1	5.73
Patrimonio %Crecimiento	-33	4.7	48
Inversionistas %	5.3	30.3	150
Mediano Plazo	**Min**	**Media**	**Max**
Inversión inicial (millones)	0.01	72.7	1000
Comisión	1.0	1.64	2.5
Rendimiento %Crecimiento	4.18	5.15	6.7
Patrimonio %Crecimiento	-63.9	-13,5	122
Inversionistas %	-54.1	-13.3	71.8
Largo Plazo	**Min**	**Media**	**Max**
Inversión inicial (millones)	1	1	1
Comisión	1.21	1.6	2.0
Rendimiento %Crecimiento	3.93	4.49	5.75
Patrimonio %Crecimiento	-72	-28	40
Inversionistas %	-41.1	-13.4	26.7
Alto Rendimiento	**Min**	**Media**	**Max**
Inversión inicial (millones)	0.2	171	534
Comisión	1.28	1.42	1.51
Rendimiento %Crecimiento	4.68	5.23	5.78
Patrimonio %Crecimiento	11.6	11.7	11.8
Inversionistas %	1.3	8.0	14.7

Elaboración propia con datos de Economatica.

Cuadro 2: Estadísticas descriptivas FICS Renta Fija

04 | Datos

Los datos para los FICs de renta fija colombianos se obtuvieron de la plataforma de información financiera Economatica, la cual provee datos sobre valor por unidad, activos netos totales, número de suscriptores, comisiones administrativas y compañía administradora. El periodo de evaluación del desempeño comprende desde enero 2010 hasta mayo de 2018 utilizando información diaria. Sin embargo, el análisis de gestión utiliza información diaria solo de 2017 debido al costo computacional.

4.1. Unidad de análisis

Nuestra unidad de análisis son los FICs de renta fija nacional, definidos por Asobolsa y Asofiduciarias (2018) como aquellos cuyo portafolio esté invertido exclusivamente en instrumentos de renta fija, donde al menos el 85% del valor del fondo proviene de (i) instrumentos de emisores locales (ii) instrumentos extranjeros emitidos en el mercado local o (iii) instrumentos de emisores locales emitidos en mercados internacionales denominados en pesos.

Asobolsa y Asofiduciarias (2018) definen unos intervalos de duración total de los fondos para determinar a qué categoría pertenece un fondo dado. Esta clasificación es fundamental para la selección de los índices de referencia utilizados en la evaluación de desempeño, ya que define una línea de acción para agrupar los fondos con una inversión pasiva en la misma categoría de riesgo.

Esta clasificación es fundamental para la selección de los índices de referencia utilizados en la evaluación de desempeño

4.2. Indices de referencia

Siguiendo el trabajo de Elton, Gruber, y Blake (1995), utilizamos los siguientes índices del mercado colombiano como factores explicativos del comportamiento de los fondos de renta fija, de esta forma, los retornos de estos índices tienen la capacidad de explicar la serie de tiempo de los retornos de los fondos:

IBR: El Indicador Bancario de Referencia refleja las condiciones de oferta y demanda de dinero del peso colombiano. Esta variable se usará como tasa libre de riesgo a corto plazo; es decir, una tasa de interés de corto plazo que refleja el precio al que los agentes están dispuestos a ofrecer o a captar recursos en el mercado monetario sin exigir una prima de riesgo.

COLTES CP: Índice de retorno total ponderado por capitalización de mercado sobre los títulos de deuda pública en pesos de Corto Plazo, desarrollado por la Bolsa de Valores de Colombia. Este índice permite seguir al mercado de renta fija colombiano para un plazo entre 1 y 5 años.

COLTES: Índice de retorno total ponderado por capitalización de mercado, mide la evolución general del segmento de títulos de deuda pública interna TES Clase en pesos, desarrollado por la Bolsa de Valores de Colombia, este índice permite seguir al mercado de renta fija colombiano a un plazo mayor que con el COLTES CP.

IDC: El índice de Deuda Corporativa diseñado por Casa de Bolsa[3] es un indicador de desempeño para una inversión promedio del mercado de deuda corporativa colombiana. Este índice permite comparar la dinámica de instrumentos de renta fija de emisores diferentes al gobierno nacional.

La figura 4 muestra las duraciones históricas de los índices propuestos para la evaluación de desempeño en las diferentes categorías, este gráfico recoge información suministrada por la Bolsa de Valores de Colombia y Casa de Bolsa.

[3] Las condiciones de rebalanceo, política de cupones y criterios de selección pueden consultarse en https://www.casadebolsa.com.co

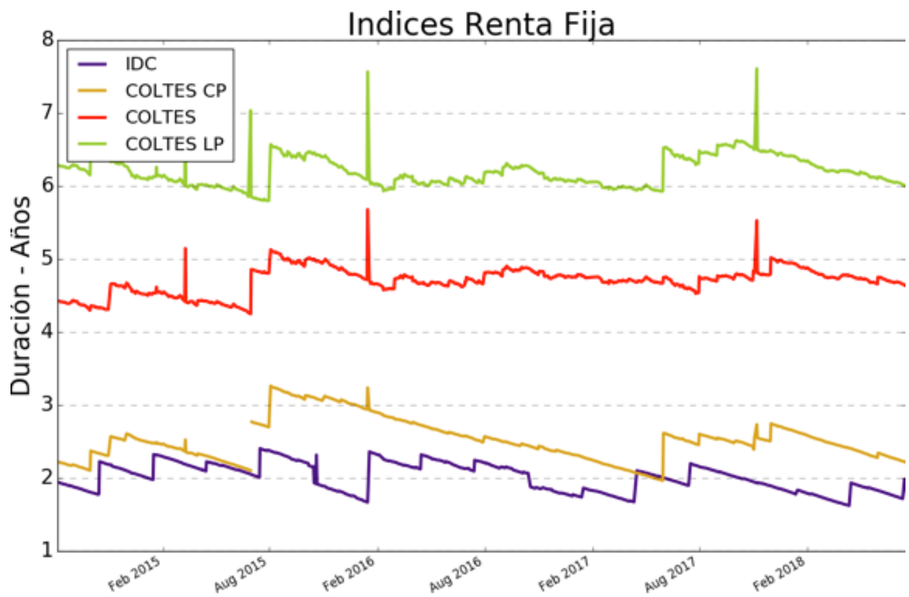


Figura 4: Duración de los índices. Fuente: BVC y Casa de Bolsa

05 | Modelos y Algoritmos

5.1. Modelos para la evaluación de desempeño

A cada categoría se le asignará como benchmark o inversión pasiva uno de los índices de referencia descritos en la sección anterior, con la condición de que su duración le permita pertenecer a dicha categoría. Así, luego de ser ambos (benchmark y fondo) instrumentos de renta fija, el hecho de tener una duración similar hará que sean comparables en riesgo de tasa de interés, permitiendo calcular el desempeño anormal por exposición al riesgo para cada fondo dada su categoría.

Específicamente:

$$R_{it}-Rf_t=\alpha_i+\beta_i\,(I_t-Rf_t)+\varepsilon_{it}$$

Donde:

R_{it}: retorno del fondo i en el periodo t

Rf_t: retorno libre de riesgo (IBR) en el periodo t,

α_i: retorno anormal promedio para el fondo i,

I_t: retorno del índice apropiado en t, y

β_i: sensibilidad del exceso de retorno del fondo i al exceso de retorno del índice

El intercepto de la regresión 1 debería ser cero para cada FIC bajo la hipótesis de mercados eficientes. Este intercepto es llamado alfa y se utilizará como medida de desempeño. Un alfa positivo o negativo, y estadísticamente significativo, sugiere un retorno anormal de dicho fondo con respecto al retorno justo dado su nivel de exposición de riesgo sistémico.

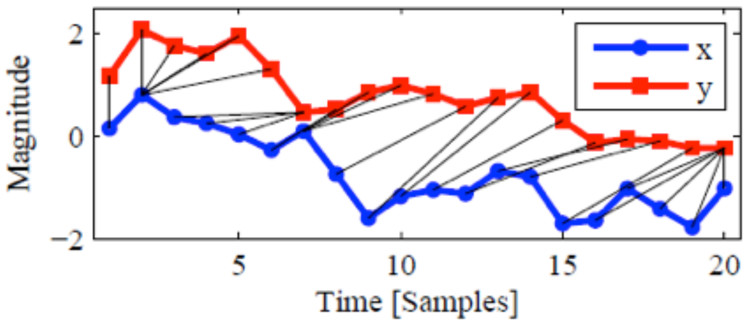
En el caso de los fondos de liquidez, el benchmark será la tasa libre de riesgo (IBR) y la especificación pasaría a ser el siguiente modelo de un único factor:

$$R_{it}-Rf_t=\alpha_i+\beta_i\,(I_t-Rf_t)+\varepsilon_{it}$$

Así, el modelo básico propuesto para evaluar el desempeño de los fondos puede entenderse como un modelo de índice único.

5.2. Algoritmos para el análisis de gestión

Para evaluar las diferencias en estilos de gestión de fondos de la misma categoría usaremos un enfoque de DTW (siglas en inglés para dynamic time warping), una metodología de programación dinámica que permite calcular el grado de disimilitud entre dos series de tiempo (Berndt y Clifford, 1994). DTW es comúnmente aplicado en el reconocimiento de voz, minería de datos, reconocimiento de gestos, robótica y medicina (Salvador y Chan, 2007). En el análisis de series de tiempo, usar DTW permitirá medir la disimilitud entre dos series, representando cada una el valor de una inversión, aún si los retornos en estas varían en el tiempo o en magnitud. El DTW funciona alineando las series de tiempo analizadas en el dominio temporal de tal forma que el costo de esta alineación sea mínimo, el objetivo de esta alineación es cuantificar su costo y, por tanto, la disimilitud entre ambas series (figura 5).



El costo acumulado puede expresarse como un problema de programación dinámica aplicando de forma recursiva:

$$D_{i,j}=f(x_i,y_j)+min\{D_{i,j-1},D_{i-1,j},D_{i-1,j-1}\}$$

Figura 5: Tomado de Serra y Arcos (2014)

Para $i=1,...,M$ y $j=1,...,N$ siendo M y N el número de observaciones para las series x e y respectivamente. La función de costo local $f(\cdot)$ será la medida de distancia entre x_i y y_j, $f(x_i,y_j)=(x_i-y_j)$ la figura 6 muestra el proceso de llenado de la matriz de costo D celda por celda hasta alcanzar la medida de disimilitud final, la entrada $D_{M,N}$. El costo acumulado total equivale a la disimilitud final, es decir a $d_{-DTW}(x,y)=D_{M,N}$ (Serra y Arcos, 2014).

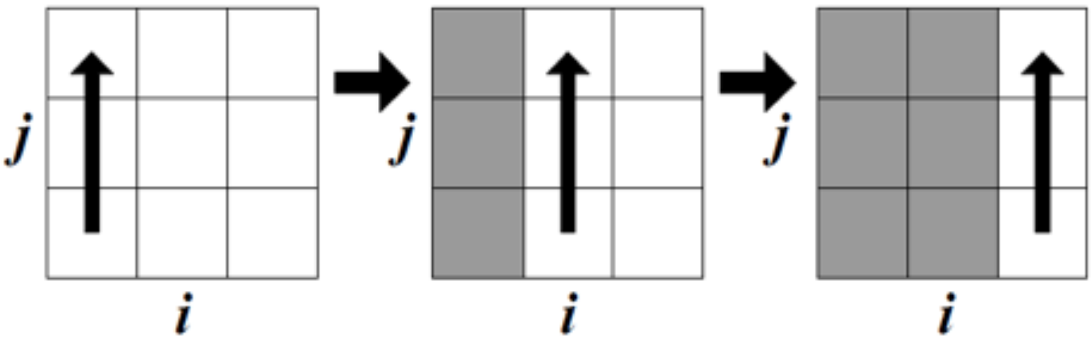


Figura 6: Tomado de Salvador y Chan (2007)

06 | Evaluación de Desempeño

Esta sección recoge los resultados de la evaluación econométrica por categoría de fondos; en todas ellas hay al menos un fondo generador de valor y los resultados agregados indican que las FICs de renta fija son, en general, instrumentos de inversión capaces de generar valor.

6.1. Fondos Renta Fija Nacional Liquidez

En esta categoría entran aquellos FICs de renta fija nacional con una política de inversión conservadora, estos fondos buscan preservar el capital invertido y poseen una duración total menor o igual a 240 días. Para los fondos de liquidez se usará como benchmark el IBR, ya que la baja duración establecida para estos fondos hace que este indicador del mercado monetario se ajuste a la categoría de riesgo por duración de los fondos.

La figura 7 muestra la evolución comparativa de 5 inversiones hipotéticas, todas por un valor inicial de COP 100 y durante un mismo horizonte de tiempo. La primera de ellas consiste en un portafolio que crece a la tasa IBR con reinversión luego de retención sobre los intereses; las 4 restantes reflejan la evolución del valor inicial de COP 100 si hubiesen sido invertidos en diferentes fondos de liquidez, usando datos reales de desempeño.

Estimando la especificación econométrica de la forma 2 obtenemos la evaluación de desempeño histórico de los fondos de liquidez. El cuadro 5 en el Apéndice contiene los resultados por fondo y clase. Los resultados indican que 7 de los 14 administradores generaron un alfa positivo y significativo en al menos una de las distintas clases ofertadas. Además, ningún fondo de liquidez posee una clase con alfa negativo y significativo. En términos globales, de las 33 clases ofertadas, 12 generaron valor de forma significativa, mientras que el alfa de las 21 clases restantes no resultó ser estadísticamente diferente de cero; es decir, ningún fondo de liquidez destruye valor.

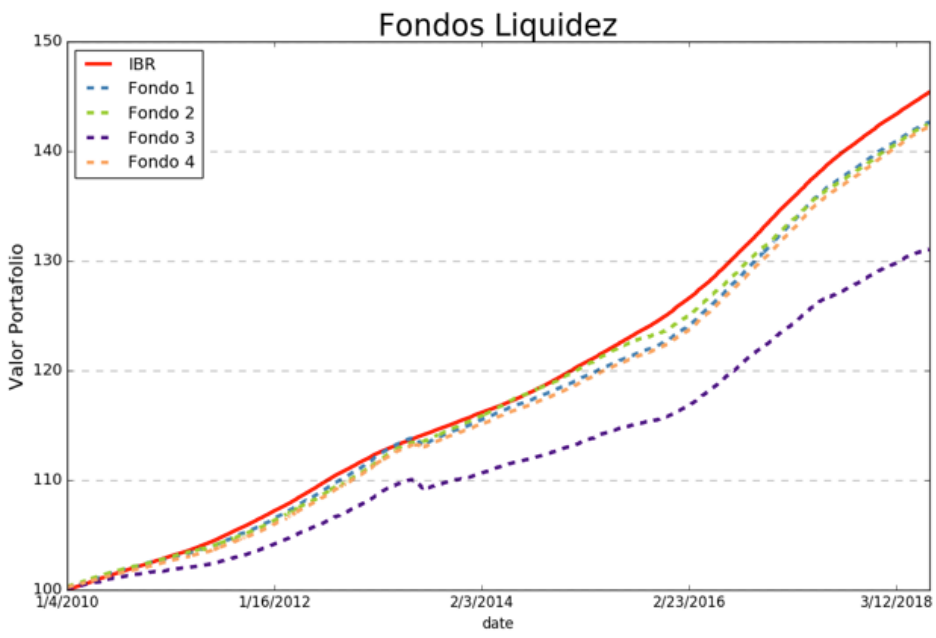


Figura 7: Desempeño 2010-2018

6.2. Fondos Renta Fija Nacional Corto Plazo

Pertenecen a esta categoría aquellos FICs que mantienen una duración total del valor del fondo entre 240 y 540 días, y al menos un 80% de su portafolio es invertido en instrumentos cuya calificación crediticia es alta, según la escala utilizada por las sociedades calificadoras en instrumentos con plazo superior a un 1 año.

Debido a que el índice COLTES CP exhibe una duración superior a la permitida en esta categoría de fondos, éste no es directamente comparable y, como alternativa, se propone un índice sintético construido de la siguiente forma:

$$I_t^{cp} = w \cdot COLTESCP_t + (1-w)\ IBR_t$$

Donde:

I_t^{cp}: Índice de referencia para fondos de corto plazo en el periodo t,

w: Peso relativo del indicador de renta fija en el índice,

Luego de analizar el resultado de simulaciones[4] para la serie de duración, se define w=0.2, con lo cual el índice sintético tendría una duración menor a la del índice.

COLTES CP y esto le permitiría pertenecer a la categoría de corto plazo en términos de duración, pues el promedio ponderado de las duraciones de los activos que componen un portafolio es una aproximación aceptable a la duración del portafolio (Fabozzi, Martellini, y Priaulet, 2006).

Estimando la especificación econométrica de la forma 1 obtenemos la evaluación de desempeño histórico de los fondos de corto plazo. La figura 8 muestra la evolución comparativa de 5 inversiones sintéticas para el mismo horizonte de inversión, partiendo nuevamente de un valor inicial de COP 100. Una de ellas muestra la evolución de la inversión a una tasa igual al rendimiento del índice sintético de corto plazo desarrollado, luego de impuestos, y las 4 restantes en fondos de renta fija de corto plazo.

El cuadro 6 del Apéndice contiene los resultados por fondo de corto plazo y clase, además de su significancia estadística. Los resultados indican que 5 de las 12 administradoras en esta categoría generaron un alfa positivo y significativo en al menos una de las distintas clases ofertadas, mientras que 4 de las 12 administradoras poseen al menos una clase con alfa negativo significativo. En términos globales, de las 21 clases ofertadas 9 generaron valor y 6 destruyeron valor. Los alfas de las 6 clases restantes no fueron estadísticamente diferentes de cero.

LA FIGURA 8 MUESTRA LA EVOLUCIÓN COMPARATIVA DE 5 INVERSIONES SINTÉTICAS PARA EL MISMO HORIZONTE DE INVERSIÓN, PARTIENDO NUEVAMENTE DE UN VALOR INICIAL DE COP 100.

[4] Las simulaciones consisten en un análisis de escenarios para w buscando aquel en el que la duración de COLTES CP, usualmente entre 2 y 3 años llegara a niveles de 1.5 años con la especificación sintética

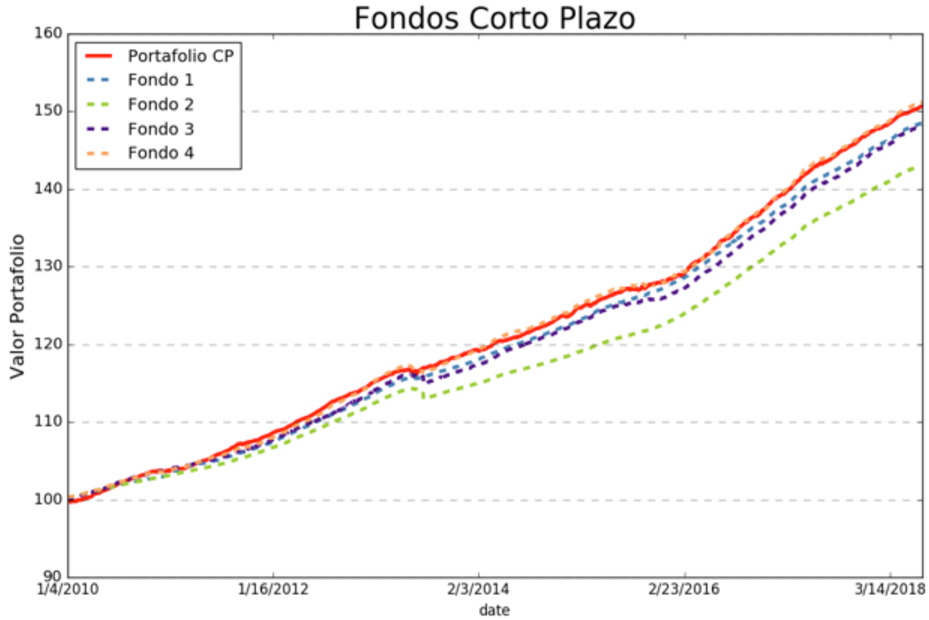


Figura 8: Desempeño 2010-2018

6.3. Fondos de Renta Fija Nacional de Mediano Plazo

En esta categoría se encuentran los fondos de inversión colectiva que buscan el crecimiento de capital en el mediano y largo plazo, con duraciones totales en un rango comprendido entre 540 y 1080 días, y al menos un 80% de su portafolio es invertido en instrumentos cuya calificación crediticia sea alta. Aunque el COLTES CP está diseñado para referir inversiones de corto plazo en los mercados de renta fija, la duración histórica de este índice usualmente entre 2 y 3 años, (véase fig 4) permite que pueda ser usado como índice de referencia para evaluar los fondos de mediano plazo ya que tienen duraciones similares.

La figura 9 muestra la evolución comparativa de 4 inversiones diferentes, una en un portafolio que crece al índice de referencia de mediano plazo y las 3 restantes en fondos de renta fija de mediano plazo. Estimando la especificación 1 obtenemos la evaluación de desempeño resumida en el cuadro 7 del Apéndice. En esta categoría, 5 fondos tienen al menos una clase que genera valor y solo un fondo destruye valor. En términos globales, 6 clases tienen alfa positivo, una tiene negativo y las 3 restantes no tienen alfa estadísticamente diferente de cero.

6.4. Fondos Renta Fija Nacional Largo Plazo

Estos fondos persiguen el crecimiento de capital en el largo plazo y tienen una mayor exposición al riesgo. Las duraciones totales del valor de estos fondos superan los 1080 días y al menos el 80% de las inversiones de estos fondos deben contar con alta calificación crediticia. El índice de referencia es un portafolio que replique una inversión en el índice COLTES y la decisión de usar este índice radica en que la duración histórica (vease fig 4) de este lo hace el más adecuado para evaluar el desempeño de inversiones con duraciones mayores a 1080 días.

La figura 10 muestra la evolución comparativa de 3 inversiones hipotéticas diferentes, una en un portafolio que crece a la tasa de rendimiento del índice de referencia de largo plazo y las 2 restantes a la de fondos de renta fija de largo plazo. El cuadro 8 en el Apéndice contiene los resultados por fondo de largo plazo además de su significancia estadística. Los resultados indican que solo 1 de los 5 administradores en esta categoría generó un alfa positivo significativo; a su vez, ningún fondo reportó de forma significativa un alfa negativo. Las 4 clases restantes no resultaron estadísticamente diferentes de cero.

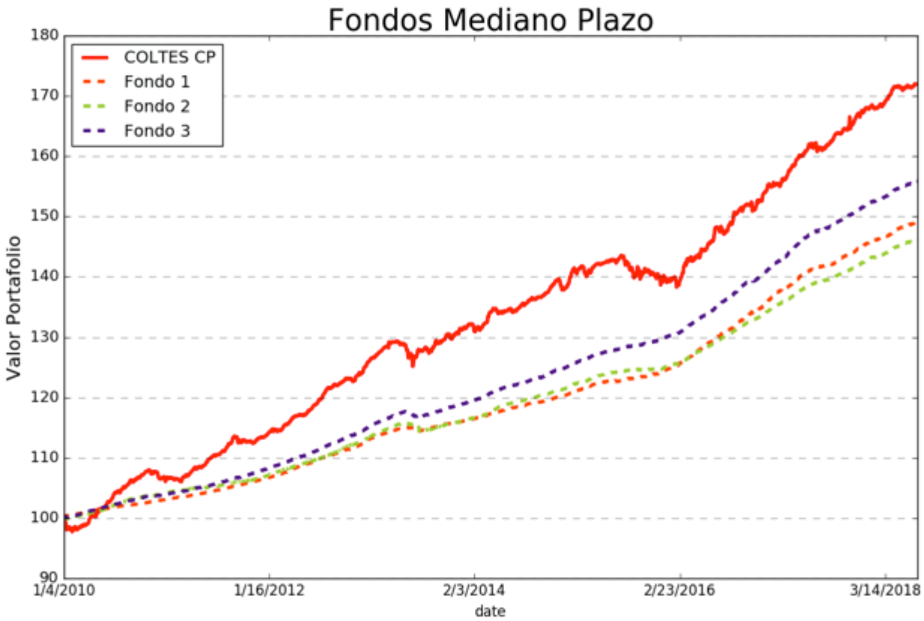


Figura 9: Desempeño 2010-2018

6.5. Fondos de Alto Rendimiento Nacional

Esta categoría cubre aquellos FICs con más de un 20% del valor invertido en instrumentos de renta fija nacional cuya calificación local sea menor o igual a la tercera más alta vigente en el largo plazo para títulos con plazo mayor a un año según la escala de las sociedades calificadoras.

Debido a que los fondos de alto rendimiento están compuestos por activos corporativos con una calificación crediticia baja, estos fondos no pueden estar en la misma categoría de riesgo que un índice que incorpore en su portafolio activos emitidos por el gobierno nacional. Así, se usará como benchmark el IDC (Índice de deuda corporativa, de Casa de Bolsa) que está conformado exclusivamente por activos corporativos de alta calificación crediticia. La figura 11 compara el valor de un portafolio invertido en IDC, es decir, el portafolio de un inversionista promedio en instrumentos de deuda corporativa, contra 4 inversiones en el mismo horizonte hechas en fondos de alto rendimiento nacional.

Luego de estimar la especificación 1 para los fondos en esta categoría, solo dos administradoras registraron resultados estadísticamente significativos (Cuadro 9 en el Apéndice): uno con alfa positivo y el otro con un alfa negativo; los demás no tuvieron un alfa estadísticamente diferente de cero.

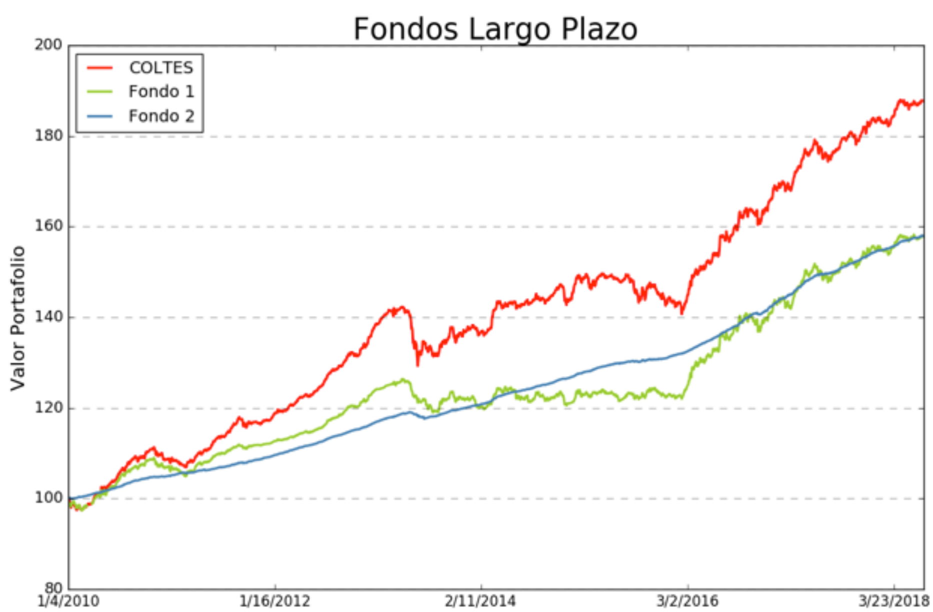


Figura 10: Desempeño 2010-2018

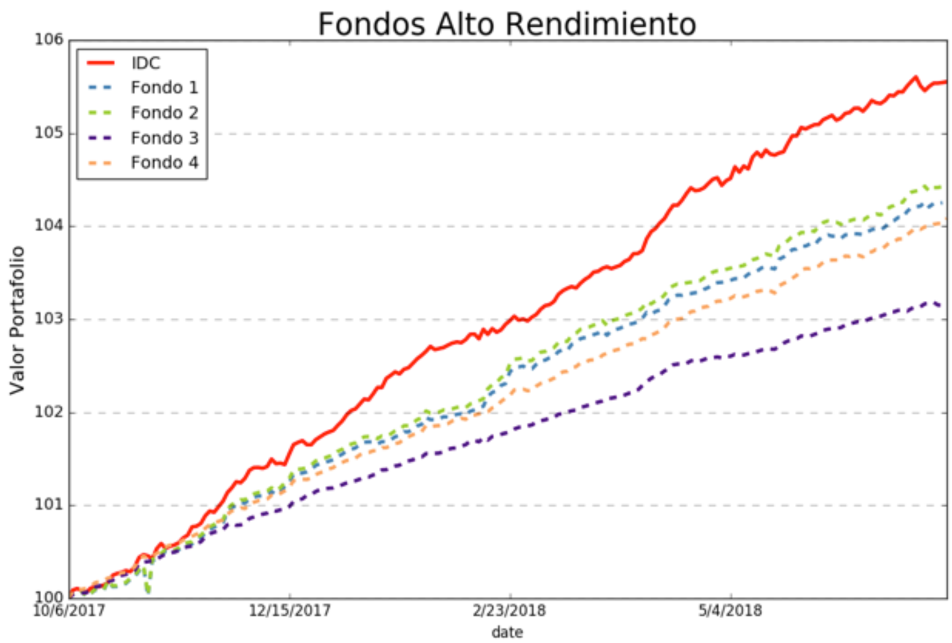


Figura 11: Datos: Economatica, BVC y Casa de Bolsa

07 | Evaluación de Desempeño

La figura 12 muestra la alineación del algoritmo DTW entre las series de una inversión en COLTES y una inversión en un fondo de largo plazo a inicios de 2017. En este trabajo se implementó este procedimiento a todas las combinaciones de fondos en una misma categoría así como al benchmark, desde el primer día hábil de 2017 hasta el último día hábil del mismo año, luego se calculó la medida de disimilitud para cada combinación usando el algoritmo en 3.

Las disimilitudes cruzadas para cada categoría, representadas como mapas de calor, se presentan en la figura 13. Cuanto más clara sea una celda más parecidos son los fondos i,j; a su vez, cuanto más intensa sea la coloración de las casillas, más disímiles entre sí son las series de tiempo de los fondos i,j. Por construcción, el benchmark de cada categoría se ubica en la última fila y columna de cada matriz.

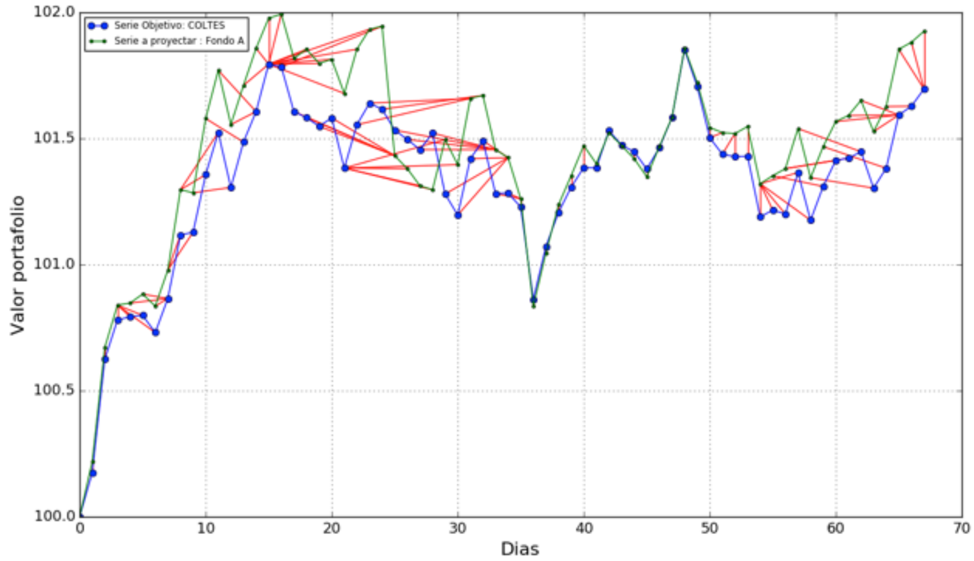
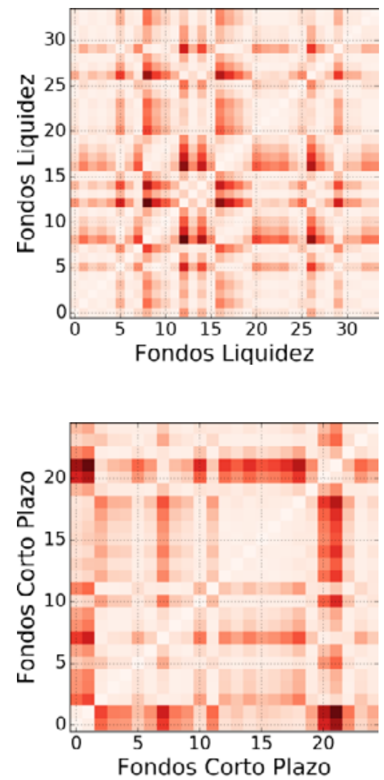
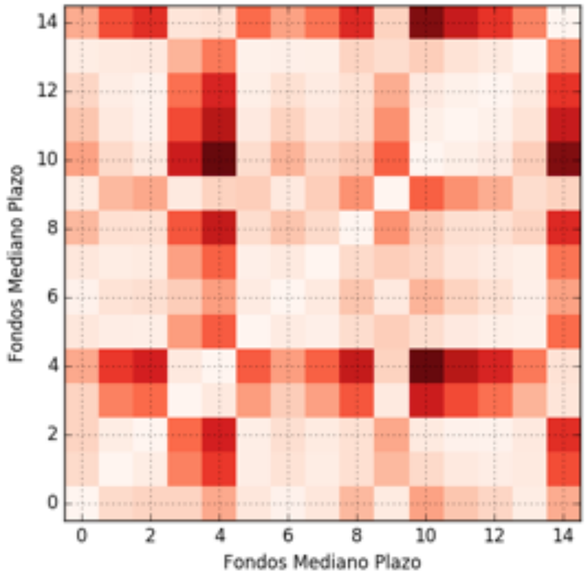
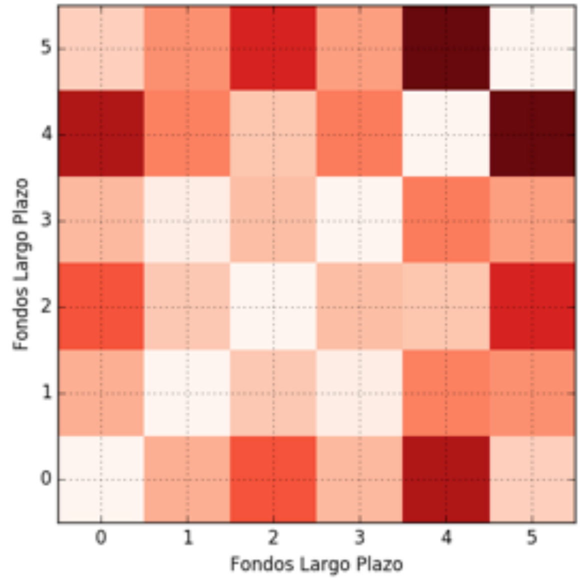


Figura 12: DTW COLTES y Fondo Largo Plazo

La figura 13 permite extraer varias reflexiones: i) La diagonal de la matriz siempre va a ser blanca, ya que la serie del valor de un fondo tiene disimilitud igual a 0 consigo misma; ii) pueden apreciarse vecindades o regiones de baja intensidad en toda categoría, esto implica que existen clusters de similitud en gestión, de acuerdo con lo cual puede decirse que existen fondos con gestión muy similar en el sentido de DTW.







De acuerdo con esto, cabría la posibilidad de explorar la relación entre las comisiones pagadas por los usuarios y el grado de similitud de los fondos, pues esto daría una idea del costo adicional asumido por invertir en fondos que finalmente no tienen resultados diferentes a otros de su cluster de vecindad.

Pasando a un enfoque cuantitativo de los resultados, el cuadro 4 compara la disimilitud promedio estandarizada entre fondos con alfa positivo y significativo contra aquellos que no tienen un alfa estadísticamente diferente de cero. El cuadro 4 sugiere que, en promedio, la disimilitud es mayor en aquellos fondos creadores de valor; es decir, aquellos fondos con alfa positiva se alejan de forma significativa en sus gestiones comparados con el resto de fondos en la misma categoría.

Cuadro3: Análisis de Disimilitud

	Fondos Liquidez		Fondos Corto Plazo	
	alfa (+)	alfa (0)	alfa (+)	alfa (0)
Disimilitud Estandarizada	0.164	0.160	0.183	0.151
	[12]	[21]	[9]	[6]
	Fondos Mediano Plazo		**Fondos Alto Rendimiento**	
	alfa (+)	alfa (0)	alfa (+)	alfa (0)
Disimilitud Estandarizada	0.258	0.230	0.160	0.148
	[7]	[6]	[1]	[4]

[Número de Observaciones]

Finalmente, la figura 14 agrega los resultados por categoría en un mismo análisis sugiriendo una relación positiva entre el grado de disimilitud y el valor generado en los fondos de renta fija nacional colombianos.

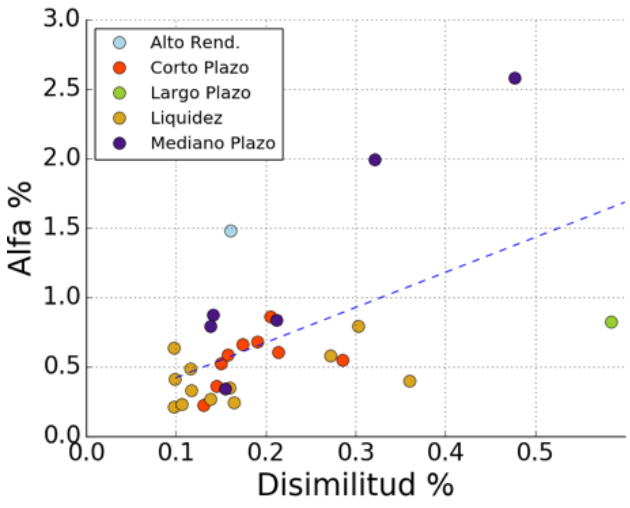


Figura 14:
Disimilitud y Desempeño

08 | Limitaciones y trabajo futuro

La principal limitación de los resultados obedece a que el mercado colombiano es aún un mercado en desarrollo, lo cual lleva a cierta escasez de información disponible para fondos y activos financieros en términos de extensión temporal. Además, el número de fondos a analizar en el mercado colombiano no es lo suficientemente grande como para extrapolar los resultados a otros mercados emergentes.

De otro lado, este trabajo abre las puertas a una investigación futura, si tenemos en cuenta que la coyuntura de mercado, el estilo de gestión y los activos que componen el fondo varían en el tiempo, se propone un análisis dinámico para medir el desempeño como una serie de tiempo, para esto se sigue un enfoque similar al utilizado por Bali y Cakici (2010):

$$R_{it}=\alpha_{it}+\beta_{it}\, I_t+\varepsilon_{it}$$

La figura 15 muestra la evolución del alfa estimado a través de la especificación 5, a la izquierda se encuentran tres fondos de corto plazo con alfa positivo y a la derecha tres fondos con alfa negativo. En promedio, anualmente los fondos a la izquierda superan en 0,9% a los demás fondos de corto plazo en periodos de alfa positivo. Por otro lado, los fondos de la derecha en períodos de alfa negativo reportan en promedio un alfa inferior en 0.085% respecto a los demás fondos de corto plazo. Es decir, los fondos con

En primer lugar, durante el período estudiado, la tasa de rentabilidad neta a 12 meses de los FICs de renta fija analizados estuvo por encima de la tasa de inflación

alfa positivos consistentes destruyen menos valor cuando todos los fondos lo hacen y viceversa, generan más valor cuando todos los fondos lo hacen.

Aunque no es objetivo de este artículo, este resultado podría ser evidencia indirecta de un efecto disposición en el mercado colombiano; es decir, la tendencia irracional de los inversionistas a vender demasiado pronto activos ganadores y a mantener activos perdedores por demasiado tiempo Odean (1998). Esta hipótesis se basa en el trabajo de Singal y Xu (2011), quienes relacionan el desempeño de los fondos con la presencia de efecto disposición en su gestión. Sus resultados encontraron que un aumento del 1% en el spread de efecto disposición reduce el alfa en 0.0123% en términos mensuales, concluyendo que fondos propensos al efecto disposición destruyen valor.

09 | Conclusiones

Aunque durante los últimos años los fondos de inversión colectiva colombianos, particularmente aquellos de renta fija, han mostrado un crecimiento importante en términos de monto administrado y número de inversionistas, su estudio a nivel académico es incipiente. Este trabajo estudia el desempeño de 48 fondos de inversión colectiva de renta fija en Colombia entre 2010 y 2018, con el fin de determinar si estos generan valor para los inversionistas y si existe una relación entre la similitud de su gestión y su desempeño. La motivación principal de este artículo es contribuir a la cantidad de información disponible para los inversionistas sobre los FICs de renta fija.

El análisis de los datos deja ver varios resultados interesantes. En primer lugar, durante el período estudiado, la tasa de rentabilidad neta a 12 meses de los FICs de renta fija analizados estuvo por encima de la tasa de inflación, de manera que estos instrumentos de inversión podrían utilizarse como un mecanismo de protección del poder adquisitivo de la moneda. En segundo lugar, ajustando un modelo de índice único, 19 de los fondos estudiados mostraron un desempeño positivo comparado con su nivel de exposición al riesgo sistémico; es decir, podrían efectivamente generar valor para los inversionistas. En tercer lugar, utilizando programación dinámica, se encontró que existen fondos gestionados de forma diferente, a juzgar por el nivel de disimilitud calculado. Finalmente, para el período de estudio, se observó evidencia de que la disimilitud entre los fondos está positivamente correlacionada con su desempeño, según lo cual podría afirmarse que son aquellos que se diferencian de otros fondos los que más generan valor para los inversionistas.

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APENDICE

Fondo	Clase	Administrador	Alfa (%)
Accival Vista	Única	Acciones y Valores S.A.	0.412***
BBVA Efectivo	Clase A	BBVA Fiduciaria S.A	0.130
Confirenta	Única	Fiduagraria S.A.	0.111
Consolidar	Única	Fiduciaria Davivienda	0.094
Efectivo A La Vista	Tradicional	Fiduprevisora	0.212**
Fic Abierto Sumar	Admon1 Admon2 Admon3 Inv. Tipo 1 Inv. Tipo 2 Inv. Tipo 3 Inv. Tipo 4 Inv. Tipo 5	Fiduciaria Bogota S.A.	-0,026 0,107 0.239* -0,105 0.020 0.030 0,125 0.40***
Efectivo	Tipo D Tipo E	Old Mutual Fiduciaria S.A.	0.487** 0.577***
Fiducuenta	Única	Fiduciaria Bancolombia S.A.	0,138
Abierto Alianza	Tipo A Tipo B Tipo C Tipo D Tipo E	Alianza Fiduciaria S.A.	0,105 0,154 0,201 0.227* 0.35**
Fonval	Única	Credicorp Capital S.A.	0,139
Liquidez	Clase A	BTG Pactual S.A.	-0,194
Liquidez	P1 P2 P3	Ultraserfinco S.A.	0,215 0,087 0,074
Occirenta	B C D	Fiduoccidente S.A.	0.792** 0.265* 0,213
Valor Plus I	Tipo 1 Tipo 2 Tipo 3 Tipo 4	Fiduciaria Corficolombiana S.A.	0,243 0.270 0.632*** 0.327*

*** p<0.01, ** p<0.05, * p<0.1

Cuadro 4: Desempeño Anualizado Fondos Liquidez

Fondo	Clase	Administrador	Alfa (%)
Avanzar Vista	Tipo 2 Tipo 3	Fiducoomeva	-0.502*** -0,059
BBVA FAM BBVA Plazo 30	Clase A Única	BBVA Fiduciaria S.A.	-0,189 0.36*
Capital Trust	Única	Itaú Asset Management	-0,233
Rendir	Única	Colpatria	-0.685***
Efectivo a Plazo	Tipo 1 Tipo 2	Previsora S.A.	-0.140* 0,013
Esparta 30	Única	Ultraserfinco S.A.	0,071
Fidurenta	Partic. 30 Partic. 60 Partic. 90 Partic. 180 Partic. 360	Fiduciaria Bancolombia S.A	0.520*** 0.582*** 0.657*** 0.677*** 0.857***
Fonval Deuda Corporativa	Única	Credicorp Capital S.A.	0.602***
Multiescala	Clase A	Corredores Davivienda	0.545***
Occibonos	Única	Fiduoccidente S.A.	0.220**
Pacto 25 Renta pais Renta plazo	Única Única Única	Fiduagraria S.A.	-0.707*** -0.94*** -0.51***
Rentar 30	Única	Fiduciaria Popular S.A.	0,189

*** p<0.01, ** p<0.05, * p<0.1

Cuadro 5: Desempeño Anualizado Fondos Corto Plazo

Cuadro 6: Desempeño Anualizado Fondos Mediano Plazo

Fondo	Clase	Administrador	Alfa (%)
Credinvertir	Credinvertir	Itaú Asset Management	0.342*
Daviplus Renta Fija	Única	Fiduciaria Davivienda	-1.39***
Deuda Corporativa	Única	Fiduciaria Corficolombiana S.A.	-0.101
Deuda Privada	Clase A Clase B	BTG Pactual S.A.	1.99*** 2.58***
Esparta 180	Única	Ultraserfinco S.A.	-0.112
FIC Fiducoldex 60 Moderado	Única	Fiducoldex S.A.	0.875***
Renta fija Pesos	Clase D	Old Mutual Fiduciaria	0,525
Fonval Deuda Pública	Única	Credicorp Capital S.A.	-0.477
Plan Semilla	Única	Fiduciaria Bancolombia S.A.	0.837***
Renta Fija 90	Tipo 1 Tipo 2 Tipo 3 Tipo 4	Alianza Fiduciaria S.A.	-0.230 0,074 0,277 0.792***

*** p<0.01, ** p<0.05, * p<0.1

Fondo	Clase	Administrador	Alfa (%)
Deuda Pública	Clase A	BTG Pactual S.A.	-1,11
Fonval Deuda Corp Mediano Plazo	Única	Credicorp Capital S.A.	1.10
Optimo	Única	Fiduciaria Bogotá S.A.	-0,847
Renta Fija Plazo	Única	Fiduciaria Bancolombia S.A.	0.825***
Helm Tesoro	Única	Itaú Asset Management	0,255

*** p<0.01, ** p<0.05, * p<0.1

Cuadro 7: Desempeño Anualizado Fondos Largo Plazo

Fondo	Clase	Administrador	Alfa (%)
Altarenta	Tipo 1 Tipo2	Fiduciaria Bogotá S.A.	1,06 1,22
Capital Plus	Única	Fiduciaria Corficolombiana S.A.	-0.29***
EspartaPlus	Única	Ultraserfinco S.A.	2,19
Renta Fija Plus	Única	Valores Bancolombia S.A.	1.48***

*** p<0.01, ** p<0.05, * p<0.1

Cuadro 8: Desempeño Anualizado Fondos Alto Rendimiento Nacional

NUMERICAL SOLUTIONS TO PDE REPRESENTATIONS OF DERIVATIVES WITH BILATERAL COUNTERPARTY RISK AND FUNDING COSTS

Torres Laserna, Nicolás

ABSTRACT

The purpose of this paper is to present numerical solutions to PDE representations for pricing derivatives including bilateral credit valuation adjustments and funding costs valuation adjustment as presented in Burgard and Kjaer (2011b). In particular, we use Crank-Nicolson finite-difference scheme to solve Black-Scholes risk-free PDE, for European and American options, and showing afterwards how this numerical solution approach is extendable to solve the PDE for the risky value of the same derivative, using the same finite-difference scheme and algorithm. Also, we present numerical solutions to valuation adjustments derived from PDE representations for European options through Monte Carlo simulation and numerical integration, and we finish by exploring an empirical approach for American options through Monte Carlo simulation, least-squares and numerical integration.

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01 | Introduction

A key form of regulation is determining the minimum amount of capital that a given bank must hold. Capital acts as a buffer to absorb losses during turbulent periods and, therefore, contributes significantly to defining creditworthiness. Ultimately, regulatory capital requirements partially determine the leverage under which a bank can operate. The danger of overly optimistic capital requirements has been often highlighted, with losses not just exceeding, but dwarfing, the capital set aside against them. Banks strive for profits and will therefore naturally wish to hold the minimum amount of capital possible to maximize the amount of business they can do and risk they are able to take (Gregory (2015)).

From 2009, new fast-tracked financial regulation started to be implemented and was very much centered on counterparty risk and OTC derivatives. The US DoddFrank Wall Street Reform and Consumer Protection Act 2009 (DoddFrank) and European Market Infrastructure Regulation (EMIR) were aimed at increasing the stability of the over-the-counter (OTC) derivative markets. The Basel III rules were introduced to strengthen bank capital bases and introduce new requirements on liquidity and leverage. The completely new credit valuation adjustment capital charge was aimed directly at significantly increasing counterparty risk capital requirements. Additionally, the G20 agreed a clearing mandate whereby all standardized OTC derivatives be cleared via central counterparties with the view that this would, among other things, reduce counterparty risk. Later, the G20 introduced rules that were to require more collateral to be posted against those OTC derivatives that could not be cleared (bilateral collateral rules) (Gregory (2015)).

The purpose of this paper is to present numerical solutions to PDE representations for pricing derivatives including bilateral credit valuation adjustments (CVA) and funding cost valuation adjustment (FVA) as presented in Burgard and Kjaer (2011b). PDE representations derived from replication arguments are in general more intuitive as they allow the relationships between cash positions to be described explicitly. Also, PDE approaches can be linked to expectations through the Feynman-Kac theorem and hence can be used to give a general formula for valuation adjustment terms. Even if the assumptions used include deterministic rates, for example, once the Feynman-Kac theorem has been applied it is relatively straight forward to generalize the resulting formulae ***(Green (2016)).***

We use Crank-Nicolson finite-difference scheme to solve Black-Scholes risk-free PDE, for European and American options, and later show how this numerical solution approach is extendable to solve the risky value PDE of the same derivative using the same finite-difference scheme and algorithm. Also, we present numerical solutions to general formulas for valuation adjustments derived from PDE representations for European options through Monte Carlo simulation and numerical integration, and finally we explore an empirical approach for American options through Monte Carlo simulation, least-squares and numerical integration. Explicit code for the solutions is provided in Appendix A.

The remainder of this paper is organized as follows. In Chapter 2 we describe the concept of collateral agreements in the context of OTC derivatives. In Chapter 3 give an overview of valuation adjustments ***(CVA and FVA).*** Chapter 4 summarizes the model framework in Burgard and Kjaer (2011b). Chapter 5 describes the solutions we propose for the PDE representations in Chapter 4. Chapter 6 presents our results. Chapter 7 contains concluding remarks and future extensions.

02 | Collateral Agreements: Credit Support Annex in the ISDA Master Agreement

OTC derivatives between two parties, the seller and the counterparty, are often documented and ruled by a Master Agreement (MA) during the life of the contract. The International Swaps and Derivatives Association (ISDA) MA is one of the most popular and widely used in the financial industry. Collateral agreements, like the credit support annex (CSA) of the ISDA MA, help to mitigate default under some scenarios by minimizing the exposure both counterparties face upon default by following certain mechanisms and conditions for collateral to be posted. This intends to replicate margin accounts in exchange traded derivatives. To understand the model framework in Chapter 4 and why it is important to determine if a single asset or portfolio of assets valuation should be adjusted by credit risk depending on the credit quality of both counterparties and the eligible collateral within an agreement, we provide an overview of the ISDA MA and some market standards regarding CSAs. As concluded by Piterbarg (2010), collateral is used to offset liabilities in case of a default, it could be thought as an essentially risk-free investment, so the rate on collateral is usually set to be a proxy of a risk-free rate such as the fed funds rate for dollar transactions, Eonia for euro, etc. Often, purchased assets are posted as collateral against the funds used to buy them, such as in the repo market for shares used in delta hedging. When collateral cannot be posted or there is counterparty risk that cannot be hedged, derivatives' valuation should reflect that risk. All the following information regarding ISDA MA and CSAs was found in Fitch Ratings (2017).

2.1 ISDA MA General Provisions

ISDA MA addresses matters such as representations and undertakings by the parties, events of default and other termination events, and payment methods and payment measures arising upon early termination. The ISDA master agreement is typically governed by either New York or English law. However, in some instances there are MAs drafted in a local language and governed under local law. Although such local master agreements can simply be a translation of an ISDA MA.

The 2002 ISDA MA is similar in form and substance to the 1992 version, with many of the substantive differences between the agreements relating to termination. Although

The 2002 ISDA MA is similar in form and substance to the 1992 version...

the events that can bring about termination have not changed materially, the time in which termination can be affected after certain events occurring has been shortened, and the payment measure for calculating payments upon termination is different.

Where more than one derivative exists between the same counterparty and seller using a single ISDA master agreement with multiple confirmation documents, there are documents for netting arrangements for termination payments and collateral posting. Where payments under the different derivatives are paid at the same position in the counterparty's priority of payments and the derivatives are concluded under the same ISDA master agreement, the documentation can provide for the netting of termination payments.

The CSA provides clarity on the collateral enforcement rights when the counterparty is the defaulting or sole affected party. Based upon the provisions of standard CSAs, the collateral amount should be calculated by a valuation agent in a commercially reasonable manner, acting in good faith and considering the prevailing market environment.

2.1.1 Derivative Documentation

The master agreement is accompanied by a schedule and a confirmation, which supplement and override to the extent of any inconsistency the master agreement. If there is an inconsistency between the schedule and the confirmation, the confirmation takes precedence. The confirmation details such as the actual rates and indices governing the relevant derivative, the dates when payments are due, and the notional amount for calculating the payments. The schedule will apply, supplement or amend certain provisions in the master agreement and will often introduce additional termination events (ATEs).

In addition, the terms of collateralization to mitigate counterparty exposure are typically set out in a CSA, in a form published by ISDA for both English and New York law. Experience with market participants suggests that agreeing and putting in place a CSA is a time-consuming exercise.

Aside from the details on the collateralization procedures, the CSA also addresses matters such as the duties of the counterparties, the frequency of the marking-to-market of collateral and derivative valuation, and the posting of collateral, the types of eligible collateral, and the minimum transfer amount in relation to a delivery or return of collateral.

2.1.2 Events of Default and Termination Events

The ISDA master agreement defines events of default (EoDs) and termination events that can bring about the early termination of a derivative. An EoD gives the non-defaulting party the right to terminate all derivative transactions under the master agreement and, where elected, may provide for automatic termination following a bankruptcy event of default. A termination event gives either one or both parties the right to terminate one or more, but not necessarily all, derivatives between them under the master agreement. The events of default set out in the ISDA master agreement can be summarized as follows:

» Failure to pay or deliver: A party fails to make any payment or due delivery, with a grace period of three business days (ISDA 1992) or one business day (ISDA 2002) after notification.

» Breach of agreement: A party fails to comply with any other obligation in accordance with the agreement, and this is not remedied within 30 days after notification.

» Credit support default: The party relies on a credit support provider and/or credit support document and there is a default with regarding this provider and/or document.

» Misrepresentation in a material respect.

» Default under a specified transaction.

» Cross-default, which is default on certain other debt over an agreed threshold amount.

» Bankruptcy or similar insolvency events.

» Merger without assumption: One party merges, and the merged entity does not assume certain obligations.

The termination events set out in the ISDA master agreement can be summarized as follows:

» Illegality: A change in the law makes it illegal for a counterparty to abide by the terms of the derivative agreement.

» Force majeure event (ISDA 2002 only): A party cannot comply due to an event of force majeure or act of state (commonly cited examples include a natural disaster, an act of terrorism or an act of war) and cannot cure the noncompliance within a specified period.

» Tax event: A change in tax law makes, or will make, a party withhold or deduct tax.

» Tax event upon merger: A party will have to withhold or deduct tax due to the merger of a party.

» A credit event upon merger: A party merges, and the merged entity is substantially weaker than before.

2.1.3 Determination of the Termination Payment Amounts

Payments upon early termination are handled differently by the 1992 and 2002 ISDA master agreements and can also receive different treatment if EoDs or termination events occur.

The 1992 ISDA master agreement provides for two payment methods (first method and second method) and two payment measures (market quotation and loss). If early termination results from an EoD, the first method provides that payments upon termination will be due only to the non-defaulting party (i.e. the defaulting party is not due any payment even if it was in the money upon termination). The second method provides that payments upon termination are due to the party in the money upon termination, regardless of whether the party is the defaulting or the non-defaulting party. The market quotation payment measure is defined as an amount determined by reference to the market for an instrument like the terminated derivative. The loss payment measure is defined as the sum of total losses and costs suffered by, or gains of, the non-defaulting party upon termination of the derivative, determined reasonably and in good faith by the non-defaulting party.

> **If early termination results from a termination event rather than an EoD**

Derivatives using the 1992 master agreement typically use the second method and market quotation. Under this arrangement, the non-defaulting party presents the derivative terms to a prescribed number of dealers that will be asked to quote a price to take over the derivative from the defaulting counterparty. If three or more quotations can be obtained, the arithmetical mean of the three quotations will be taken, and the party that is out of the money will have to pay that amount to the party that is in the money. There will also be an account taken of any unpaid amounts that arise on or before the date of termination.

If early termination results from a termination event rather than an EoD, the course of action depends on whether one or both parties have been affected. If there is one affected party, the payment method is identical to the second method, regardless of whether the schedule calls for the first or second method. The payment measure applied will be market quotation or loss, as set out in the schedule. The affected party is treated as the defaulting party and the party that is not affected as the non-defaulting party for both payment method and payment measure.

If both parties are affected and market quotation applies, each party obtains a settlement amount through the market quotation methods previously described, and the payment amount is equal to half of the difference of the two results. If both parties are affected and loss applies, each party calculates its loss because of the derivatives termination, and the payment amount is equal to half of the difference of the two results.

The 2002 ISDA master agreement handles early termination payments in a slightly different manner. Payment methods and payment measures do not have to be set out in the schedule, as the agreement calls for the same payment method and payment measure in all events.

If early termination arises by an EoD, the non-defaulting party determines the close-out amount. This is essentially the amount of losses or costs or gains of the non-defaulting party in replacing, or in providing to the non-defaulting party the economic equivalent of the material terms of the derivative. To calculate this, the non-defaulting party can use information such as third-party quotations and relevant market data. As with the second method previously described, payment could be due to either the defaulting or the non-defaulting party because of this calculation. There might also be an account taken of any unpaid amounts that arise on or before the date of termination.

If early termination results from a termination event, and if there is one affected party, the calculation could be handled as with an EoD, whereby the affected party is treated as the defaulting party and the party that is not the affected party as the non-defaulting party. If both are affected, each party must calculate an amount in accordance with the paragraph above, and the payment amount is equal to half of the difference of the two results.

03 | Credit Valuation Adjustment (CVA) and Funding Valuation Adjustment (FVA)

In this chapter we briefly describe the origin and motivation of derivatives valuation adjustments (xVAs). For more information on this topic the reader may refer to Alavian et al. (2008), Green (2016), Gregory (2015), Piterbarg (2010), Brigo et al. (2009), Brigo and Capponi (2009).

CVA has become a key topic for banks in recent years due to the volatility of credit spreads and the associated accounting (e.g. IFRS 13) and capital requirements (Basel III). However, note that whilst CVA calculations are a major concern for banks, they are also relevant for other financial institutions and corporations that have significant amounts of OTC derivatives to hedge their economic risks. Indeed, CVA (and Debt Valuation Adjustment-DVA) should only be ignored for financial reporting if they are immaterial which is not the case for any significant OTC derivative user (Gregory (2015)).

Although not entirely driven by the recent financial crisis, IFRS 13 accounting guidelines were introduced from 2013 to replace IAS 39 and FAS 157. IFRS 13 provided a single framework for the guidance around fair value measurement for financial instruments and started to create convergence in practices around CVA. IRFS 13 (like the FAS 157) uses the concept of exit price, which implies the use of market-implied information as much as possible. This is particularly important in default probability estimation, where market credit spreads must be used instead of historical default probabilities. Exit price also introduces the notion of own credit risk and leads to DVA as the CVA charged by a replacement counterparty when exiting a transaction (Gregory (2015)).

Derivatives can be both assets and liabilities. When they are assets they create funding costs, but as liabilities they provide funding benefits. Transactions with large CVA (or xVAs) components are also likely to have significant funding components. In some sense, FVA is not a particularly new concept. Prior to the global financial crisis, LIBOR was used to discount cash flows: not because it was the risk-free rate (which in any case is a theoretical construct), but because it was a good approximation of a banks unsecured funding costs that were considered short-term. Post-crisis, banks have realized that they cannot be as reliant on short-term funding or fund at LIBOR and have therefore sought to incorporate these higher costs through FVA (Gregory (2015)).

FVA, like CVA, is predominantly considered for uncollateralized transactions. However, since no collateralization is perfect, it will also be a component for collateralized ones (although in some cases this may be neglected). FVA was not considered prior to 2007 because unsecured funding for institutions, such as banks, was trivial, and could be achieved at approximately risk-free rate. (Bank credit spreads were typically only a few basis points prior to 2007. but since then have been more in the region of hundreds of basis points.) This means that transactions, especially those that are uncollateralized, are now typically treated including the party's own funding as a component of their price. This is the role of FVA, although its use in accounting statements has been more controversial. From a quantification point of view, FVA is similar in many ways to CVA, and many of the components to calculate the two are the same (Gregory (2015)).

Despite the increased use of collateral, a significant portion of OTC derivatives remain uncollateralized. This arises mainly due to the nature of the counterparties involved, such as corporates and sovereigns, without the liquidity and/or operational capacity to adhere to frequent collateral calls. In general, funding costs (and benefits) in derivatives portfolios be arising from the following situations (Gregory (2015)):

» Undercollateralization. Transactions that are undercollateralized give rise to funding costs and benefits. This includes completely undercollateralized (no CSA) but also cases of partial collateralization (e.g. a two-way CSA with a material threshold). One-way CSAs are also a special case, since one party is collateralized whilst the other is not.

» Non rehypothecation and segregation. Even if a party can receive collateral, there is a question of whether this collateral can be used. If the collateral cannot be rehypothecated and/or must be segregated, this will deem it useless from a funding point of view.

There are essentially two types of models for CVA: unilateral models that only consider the credit risk of the counterparty and bilateral models that consider the credit risk of both counterparty and self. Equation (3.1) is the definition of CVA in both cases. Funding costs add further complexity. In the case of bilateral models, it is useful to write

$$U = CVA + DVA + FVA \quad (3.1)$$

where CVA is a cost and DVA is a benefit. Bilateral CVA models naturally provide two terms, a term that reduces accounting value due to counterparty risk and a term that increases accounting value due to risk of own default.

In the xVA literature (e.g. Gregory (2015) and Green (2016)) the value of a derivative can be written as

$\hat{V}$ (credit risky) = V (default free) + U(valuation adjustment)

V = Unadjusted value,i.e. Black-Scholes

$\hat{V}$ = Economic value including adjustments

U = Valuation adjustments, as in equation (3.1)

This formula highlights that CVA is the adjustment to the underlying price of the derivative. The full value of the derivative should include the impact of credit risk (Gregory (2015)).

Throughout this document we will talk indifferently about CVA as the sum of CVA and DVA in the context of a derivative contract with bilateral counterparty risk as mentioned above.

04 | Model Framework: PDE Representations of Derivatives with Bilateral Counterparty Risk (CVA) and Funding Costs (FVA)

In this chapter we briefly present the work developed by Burgard and Kjaer (2011b), which is the central axis of this present document. In the paper, the authors combine the effects of the seller's credit on its funding costs with the effects on the bilateral counterparty risk into a unified framework. Using hedging arguments, an extended Black-Scholes partial differential equation (PDE) is derived in the presence of bilateral counterparty risk in a bilateral jump-to-default model, including funding considerations in the financing of the hedge positions. Two rules are considered for the determination of the derivative mark-to-market value at default, namely, the total risky value and the counterparty-risk-free value. Content in this chapter follows closely Burgard and Kjaer (2011b) and it is presented in the body of this document for academic purposes and sake of completeness. A relevant paper to understand previous efforts to the derivation of this framework can be found in Piterbarg (2010).

4.1 Definitions and Assumptions

A derivative contract price function $\hat{V}$ is considered on asset S between seller B and a counterparty C that may both default. The asset S is not affected by a default of either B or C. Similarly, it is denoted as V the same derivative price function between two parties that cannot default. At default of either the counterparty or the seller, the value of the derivative to the seller $\hat{V}$ is determined with a mark-to-market rule M, which may be equal to $\hat{V}$ or V (throughout Burgard and Kjaer (2011b) positive derivative values correspond to seller assets and counterparty liabilities).

An economy with the following four traded assets is considered:

P_R: default risk-free zero-coupon bond.
P_B: default risky, zero-recovery, zero-coupon bond of party B.
P_C: default risky, zero-recovery, zero-coupon bond of party C.
S: spot asset with no default risk.

Both risky bonds P_B and P_C pay 1 at some future time T if the issuing party has not defaulted, and 0 otherwise. It is mentioned in Burgard and Kjaer (2011b) that these simplistic bonds are useful for modelling and can be used as building blocks for more complex corporate bonds, including those with nonzero recovery. It is assumed that the processes for assets P_R, P_B, P_C and S, under the historical probability measure, are specified by:

$$\frac{dP_R}{P_R} = r(t)dt \qquad \frac{dP_b}{P_b} = r_B(t)dt - dJ_B$$

$$\frac{dP_c}{P_c} = r_c(t)dt - dJ_c \qquad \frac{dS}{s} = \mu(t)dt + \sigma(t)dW \tag{4.1}$$

where W(t) is a Wiener process, and $\mu(t) > 0$, $r(t) > 0$, $rB(t) > 0$, $rC(t) > 0$, $\sigma(t) > 0$ are deterministic functions of t, and where J_B and J_C are two independent point processes that jump from zero to one on default of B and C, respectively. This assumption implies that a hedging strategy could be achieved using bonds P_B and P_C alone. The hedging strategy will be described in the next section.

A PDE is derived for the general case of $M(t,S)$ and two special cases where $M(t,S) = V\hat{}(t,S,0,0)$ and $M(t,S) = V(t,S)$ are considered. Let $R_B \in [0,1]$ and $R_C \in [0,1]$ denote the deterministic recovery rates on the derivative positions of parties B and C, respectively. From the above we have the following boundary conditions:

$\hat{V}(t,S,1,0) = M+(t,S) + R_B M-(t,S)º$

(seller defaults first) (4.2)

$\hat{V}(t,S,0,1) = R_C M+(t,S) + M-(t,S)$

(counterparty defaults first),

Rate	Definition	Choices discussed
r	Risk-free rate	
r_B	Yield on recoveryless bond of seller B	
r_C	Yield on recoveryless bond of counterparty C	
λ_B	$\lambda_B \equiv r_B - r$	
λ_C	$\lambda_C \equiv r_C - r$	
r_F	Seller funding rate for borrowed cash on seller's derivatives replication cash account	$r_F = r$ if derivative can be used as collateral; $r_F = r + (1 - R_B)\lambda_B$ if derivative cannot be used as collateral
s_F	$s_F \equiv r_F - r$	
γ_S	Continuous dividend yield	
q_S	Cost of financing that depends on r and the repo rate of S	
R_B	Recovery on derivative mark-to-market value in case seller B defaults	
R_C	Recovery on derivative mark-to-market value in case counterparty C defaults	

Table 4.1: Rates, spreads and recoveries

4.2 The Model

As in the classic Black-Scholes framework, the position on the derivative is hedged through a self-financing portfolio that covers all the underlying risk factors of the model. The portfolio ∏ that the seller sets up consists of $\delta(t)$ units of S, $\alpha B(t)$ units of P_B, $\alpha_C(t)$ units of P_C and $\beta(t)$ units of cash, such that the portfolio value at t hedges out the value of the derivative contract to the seller, i.e., $\hat{V}(t) + \Pi(t) = 0$. Thus:

$$-\hat{V}(t) = \Pi(t) = \delta(t)S(t) + \alpha_B(t)P_B + \alpha_C(t)P_C + \beta(t) \quad (4.3)$$

It is noted that when $\hat{V} \geq 0$ the seller will incur in a loss at counterparty default. To hedge this loss, P_C needs to be shorted, so it is expected that $\alpha C \leq 0$. If the seller can borrow the bond P_C close to the risk-free rate r through a repurchase agreement, the spread λ_C between the rate r_C on the bond and the cost of financing the hedge position in C can be approximated to $\lambda_C = r_C - r$. Since we defined P_C to be a bond with zero recovery, this spread corresponds to the default intensity of C.

On the other hand, if $\hat{V} \leq 0$, the seller will gain at its own default, which can be hedged by buying back P_B bonds, so it is expected that $\alpha_B \geq 0$. For this to work, it is needed to ensure that enough cash is generated and that any remaining cash (after purchase of P_B) is invested in a way that does not generate additional credit risk for the seller, i.e., any remaining positive cash generate yield at the risk-free rate r.

Imposing that the portfolio $\Pi(t)$ is self-financing implies that:

$$- d\hat{V}(t) = \delta(t)dS(t) + \alpha_B(t)dP_B + d\alpha_C(t)dP_C + d\bar{\beta}(t)$$

where the change in cash $d\bar{\beta}$ may be decomposed as $d\bar{\beta}(t) = d\beta_S(t) + d\beta_F(t) + d\beta_C(t)$ with:

$d\beta_S(t)$: the share position provides a dividend income of $\delta(t)\gamma_S(t)S(t)dt$ and a financing cost of $-\delta(t)q_S(t)S(t)dt$, so $d\beta_S = \delta(t)(\gamma_S(t)-q_S(t))S(t)dt$. The value of $q_S(t)$ depends on the risk-free rate and de repo rate of S(t).

$d\beta_F(t)$: From the above analysis, any surplus cash held by the seller after the own bonds have been purchased must earn the risk-free rate r(t) in order not to introduce any further credit risk to the seller. If borrowing money, the seller needs to pay the rate $r_F(t)$. For this rate there are two cases: where the derivative itself can be posted as collateral for the required funding and no haircut is assumed then $r_F(t) = r(t)$. If the derivative cannot be used as collateral, funding rate is set to the yield of the unsecured seller bond with recovery R_B: i.e. $r_F(t) = r(t)+(1-R_B)\lambda_B$. In practice the latter case is often the more realistic one. Keeping r_F general:

$$d\beta_F(t) = \{r(t)(-\hat{V} - \alpha_B P_B)^+ + r_F(t)(-\hat{V} - \alpha_B P_B)^-\}dt \quad (4.5)$$

$$= r(t)(-\hat{V} - \alpha_B P_B)dt + s_F(t)(-\hat{V} - \alpha_B P_B)^- dt \quad (4.6)$$

where the funding spread $s_F \equiv r_F - r$: i.e $sF = 0$, if the derivative can be used as collateral, and $sF = (1 - R_B)\lambda_B$ if it cannot.

$d\beta_C(t)$: By the arguments above, the seller will short the counterparty bond through a repurchase agreement and incur financial costs of $d\beta_C(t) = -\alpha_C(t)r(t)P_C(t)dt$ if zero haircut is assumed.

For simplicity the t notation is dropped. From the above, it follows that the change in the cash account (including contributions due to rebalancing at the end of the period dt) is given by:

$$d\bar{\beta} = \delta(\gamma_S - q_S)Sdt + \{r(-\hat{V} - \alpha_B P_B) + s_F(t)(\hat{V} - \alpha_B P_B)^-\} dt - r\alpha_C P_C dt \quad (4.7)$$

Now (4.4) becomes:

$$-d\hat{V} = \delta dS + \alpha_B dP_B + \alpha_C dP_C + d\bar{\beta}$$
$$= \delta dS + \alpha_B P_B(r_B dt - dJ_B) + \alpha_C P_C(r_C dt - dJ_C) \quad (4.8)$$

$$+\{r(-\hat{V} - \alpha_B P_B) + sF(-\hat{V} - \alpha_B P_B)^- - \alpha_C r P_C - \delta(q_S - \gamma_S)S\}dt$$
$$= \delta dS - \alpha_B P_B dJ_B - \alpha_C P_C dJ_C + \{\alpha_B P_B(r_B t - r) + \alpha_C P_C(r_C - r) - \hat{V}r \quad (4.9)$$

$$+s_F(-\hat{V} - \alpha_B P_B)^- - \delta(q_S - \gamma_S)S\}dt$$

By It´o's Lemma for jump diffusion and the assumption that simultaneous jump to default is a zero-probability event, the derivative value moves by

$$d\hat{V} = \partial_t \hat{V}\,dt + \partial_S \hat{V}\,dS + \frac{1}{2}\sigma^2 S^2 \partial_{SS}^2 \hat{V}\,dt + \Delta\hat{V}_B\,dJ_B + \Delta\hat{V}_C\,dJ_C, \quad (4.10)$$

where,

$$\Delta\hat{V}B = \hat{V}(t,S,1,0) - \hat{V}(t,S,0,0),$$
$$\Delta\hat{V}C = \hat{V}(t,S,0,1) - \hat{V}(t,S,0,0), \quad (4.11)$$

which can be computed from the boundary condition (4.2).

Replacing $d\hat{V}$ in (4.9) by (4.10) shows that all risks in the portfolio can be eliminated by choosing δ, α_B, α_C as

$$\delta = -\partial_S \hat{V}, \quad (4.12)$$

$$\alpha_B = \frac{\Delta\hat{V}_B}{p_B} = \frac{-\hat{V} - (M^+ + R_B M^-)}{p_B} \quad (4.13)$$

$$\alpha_c = \frac{\Delta\hat{V}_c}{p_c} = \frac{\hat{V} - (R_C M^+ + M^+)}{p_c} \quad (4.14)$$

Hence, the cash account evolution (4.6) can be written as

$$d\beta_F = \{rR_B M^- - r_F M+\}dt, \quad (4.15)$$

the amount of cash deposited by the seller at the risk-free rate equals $-R_B M^-$ and the amount borrowed at the funding rate r_F equals $-M^+$.

The following parabolic differential operator At is introduced

$$A_t V \equiv \frac{1}{2} \sigma^2 S^2 \partial_{SS}^{\,2} V + (q_S - \gamma_S) S \partial_S V, \quad (4.16)$$

hen it follows that $\hat{V}$ is the solution to the PDE

$$(\partial_t \hat{V} + A_t \hat{V} - r\hat{V} = s_F (\hat{V} + \Delta \hat{V}_B)^\wedge + - \lambda_B \Delta \hat{V}_B - \lambda_C \Delta \hat{V}_C, \ \hat{V}(T,S) = H(S) \mapsto (\text{derivative payoff function}), \quad (4.17)$$

where $\lambda_B \equiv r_B - r$ and $\lambda_C \equiv r_C - r$. Inserting (4.11) with boundary condition (4.2) into (4.17) gives

$$(\partial_t \hat{V} + A_t \hat{V} - r\hat{V} = (\lambda_B + \lambda_C)\hat{V} + s_F M^+ - \lambda_B (R_B M^- + M^+) - \lambda_C (R_C M^+ + M^-), \hat{V}(T,S) = H(S), \quad (4.18)$$

where $(\hat{V} + \Delta \hat{V}_B) + = (R_B M^- + M^+) = M^+$ was used.
In contrast, the risk-free value V satisfies the regular Black-Scholes PDE

$$\partial_t V + A_t V - rV = 0, \quad (4.19)$$
$$V(T,S) = H(S),$$

as Burgard and Kjaer (2011b) interprets λ_B and λ_C as effective default rates (intensity of default) the differences between (4.18) and (4.19) are as follows:

» The first term on the right side of (4.18) is the additional growth rate the seller B requires on the risky asset $\hat{V}$ to compensate for the risk that default of either the seller or the counterparty will terminate the derivative contract.

» The second term is the additional funding cost for negative values of the cash account of the hedging strategy.

» The third term is the adjustment in growth rate that the seller can accept because of the cash flow occurring at own default.

» The fourth term is the adjustment in growth rate that the seller can accept because of the cash flow occurring at counterparty default.

Terms one, three and four are related to counterparty risk whereas the second term represents the funding cost. From this interpretation it follows that the PDE for a so called extinguisher trade, whereby it is agreed that no party gets anything at default, is obtained by removing terms three and four from PDE (4.18).

4.2.1 Main Results of Burgard and Kjaer (2011b)

Finally, we outline the main results in Burgard and Kjaer (2011b) and pay special attention to results 2 and 3, which will be used in Chapter 5.

Main result 1: non-linear PDE for $\hat{V}$ when M = $\hat{V}$

$$\partial_t \hat{V} + A_t \hat{V} - r \hat{V} = (1 - R_B)\lambda_B \hat{V}^- + (1 - R_C)\lambda_C \hat{V}^+ + s_F \hat{V}^+, \quad (4.20)$$
$$\hat{V}(T,S) = H(S),$$

Main result 2: linear PDE for $\hat{V}$ when M = V

$$\partial_t \hat{V} + A_t \hat{V} - (r + \lambda_B + \lambda_C) \hat{V} = -(R_B\lambda_B + \lambda_C) V^- - (\lambda_B + R_C\lambda_C) V^+ + s_F V^+,$$
$$\hat{V}(T,S) = H(S), \quad (4.21)$$

Main result 3: integral equation for U when M = V

As pointed by Burgard and Kjaer (2011b), is common in the xVA literature to find the value of a risky derivative $\hat{V}$ decomposed in the risk-free value of the contract and the xVA or adjustments as

$$\hat{V} = V + U.$$

If this decomposition is inserted into (4.21) and using Black-Scholes regular PDE representation in (4.19), the U can be represented by the following linear PDE:

$$\partial_t U + A_t U - (r + \lambda_B + \lambda_C)U = (1 - R_B)\lambda_B V^- + (1 - R_C)\lambda_C V^+ + s_F V^+,$$
$$U(T,S) = 0 \rightarrow \text{(boundary condition implies no default risk at maturity)}, \quad (4.22)$$

and using the Feynman-Kac formula (see Feynman-Kac formula (2017) or Karatzas and Shreve (1998) for derivation), that states the relation between parabolic PDEs and stochastic processes, the solution U can be written as expected value (4.23) and one step ahead in (4.24) as presented in Burgard and Kjaer (2011b).

$$U(t,S)=\mathbb{E}_t\left[-\int_t^T e^{-\int_t^u r(\tau)+\lambda_B(\tau)+\lambda_C(\tau)d\tau}(1-R_B)\,\lambda_B(u)\,V^-(u,S(u))du\right]$$
$$+\mathbb{E}_t\left[-\int_t^T e^{-\int_t^u r(\tau)+\lambda_B(\tau)+\lambda_C(\tau)d\tau}(1-R_C)\,\lambda_V(u)\,V^+(u,S(u))du\right]$$
$$+\mathbb{E}_t\left[-\int_t^T e^{-\int_t^u r(\tau)+\lambda_B(\tau)+\lambda_C(\tau)d\tau}s_F(u)\,V^+(u,S(u))du\right] \quad (4.23)$$

$$\begin{gathered}U(t,S)=-(1-R_B)\int_t^T \lambda_B D_r+\lambda_B+\lambda_C E_t[V^-(u,S(u))]du \\ -(1-R_C)\int_t^T \lambda_C D_r+\lambda_B+\lambda_C\ E_t[V^+(u,S(u))]du \\ -\int_t^T s_F D_r+\lambda_B+\lambda_C)\,E_t[V^+(u,S(u))]du, \\ D_k(t,u)\equiv exp\{-\int_t^u k(v)dv\}\mapsto \\ \textit{discount factor between time t and u}\end{gathered} \quad (4.24)$$

For some cases (e.g. plain vanilla options or interest rate derivatives) the value of V can be represented by a closed-form formula, making it easier to compute the integrals in (4.23). In other cases (e.g. exotic options) these integrals must be computed numerically as analytic solutions does not exist or have not been found.

05 | Numerical Solutions to PDEs for Derivatives with CVA and FVA

In this chapter we present numerical solutions to main results 2 and 3 in section 4.2.1. For linear PDE in result 2, Crank-Nicolson finite-difference scheme is described, and pseudocode is provided, specifically for European and American options with deterministic functions for interest rates. Result 3 is solved for European options through Monte Carlo (MC) simulation of asset price and numerical integration. Also, we performed an empirical exercise for the valuation of American options with MC simulation and least-squares to estimate the conditional expected value from continuation. Explicit R (2016) code for each solution is provided in Appendix A.

Crank-Nicolson (CN) scheme is a popular finite-difference scheme among practitioners and in quantitative finance literature.

5.1 Crank-Nicolson Finite-Difference Scheme for PDEs

Crank-Nicolson (CN) scheme is a popular finite-difference scheme among practitioners and in quantitative finance literature. It is known to have better results regarding stability and convergence than explicit finite-difference method, and to have higher convergence rates to the solution of PDEs. It is an implicit finite-difference method that takes the average of explicit finite-difference method (forward-difference approximation to the time partial derivative) and implicit method (time-backward difference approximation) (Wilmott et al. (1995) and Wilmott (2006)). CN method error is $O((\Delta t)^2,(\Delta S)^2)$ and temporal or spatial mesh spaces have lower impact in the stability and convergence of the solution, relative to other finite difference schemes (Wilmott (2006)).Analysis of the efficiency, stability and convergence of CN finite-difference scheme are beyond the scope of this paper. The reader may refer for more information on this subject to Duffy (2006), Thomas (1998) and Thomas (1999). For a famous critique to CN method with valuable error fixing insights or alternative methods see Duffy (2006) and Duffy (2004). Other suggested literature for finite-difference methods is LeVeque (2007) and Thomas (1998).

In 5.1.1 we show CN scheme for the classic Black-Scholes PDE and in 5.1.2 how scheme and algorithm in 5.1.1 is modified to solve result 2 in 4.1.2, a PDE for a derivative's value with CVA and FVA.

5.1.1 CN scheme for Black-Scholes PDE for European and American options

Regular risk-free Black-Scholes PDE for an European derivative, presented in chapter 4, can be written as:

$$\mathcal{L}V=0 \text{ ($\mathcal{L}$ is a linear differential operator),} \quad (5.1)$$

with boundary condition $V(T,S) = H(S)$

which is a parabolic linear PDE. We will keep this simplified representation of the PDE in mind for later comparison with the risky value of the derivative in subsection 5.1.2. We now introduce the CN scheme to solve equation (5.1).

The temporal domain [0,T] is divided in a finite number of mesh points $0 = t_0 < t_1 < t_2 < \dots < t_{m-1} < t_m = T$ and, similarly, spatial domain [0,S] is represented by Smin = $S_0 < S_1 < S_2 < \dots < S_{N-1} < S_N = S_{max}$.

In our scheme we use uniform mesh spaces, as suggested by Duffy (2006), to preserve second-order precision of the CN method. Consider the following:

$$\Delta t= \frac{T}{m} ,t\_j=j\Delta t,j=0,\dots,m \tag{5.2}$$

$$\Delta S= \frac{S\,max\text{-}S\,min}{n} ,S_i=S_{min}+i\Delta S,i=0,\dots,N$$

Approximations of V are taken at the half step $t+\frac{\Delta t}{2}$. It follows that the representation of the partial derivatives with respect time and space for the CN scheme are as follows (see Wilmott (2006)):

$$\partial_t V= \frac{V_{i,j+1}\text{-}V_{i,j}}{\Delta t} +O((\Delta t)^2), \tag{5.3}$$

the partial derivative of V with respect to the asset price:

$$\partial_S V= \frac{V_{i+1,j}\text{-}V_{i-1,j}+V_{i+1,j+1}\text{-}V_{i-1,j+1}}{4\Delta S} +O(\Delta S^2), \tag{5.4}$$

and the second-order partial derivative of V with respect to the asset price:to the asset price:

$$\partial_{SS}^{\,2} V= \frac{V_{i+1,j}\text{-}2V_{i,j}+V_{i-1,j}+V_{i+1,j+1}\text{-}2V_{i,j+1}+V_{i-1,j+1}}{2\Delta S^2} +O(\Delta S^2), \tag{5.5}$$

and if we set aside error terms O(.) and replace (5.3), (5.4) and (5.5) in (4.19), setting S_{min}=0, we obtain the finite-difference representation of Black-Scholes PDE in the form:

$$\frac{V_{i,j+1}-V_{i,j}}{\Delta t}$$

$$+\sigma_j^2 i_j^2 \frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}+V_{i+1,j+1}-2V_{i,j+1}+V_{i-1,j+1}}{4}$$

$$+(q_{Sj}-\gamma_{Sj})\, i \frac{V_{i+1,j}-V_{i-1,j}+V_{i+1,j+1}-V_{i-1,j+1}}{4} \quad (5.6)$$

$$-r_j \frac{V_{i,j+1}+V_{i,j}}{2} = 0.$$

As we have defined our spatial mesh points, we will work backwards in time and from set boundary conditions we take advantage of the fact that we know the value of the derivative at expiry T, so it is convenient to rearrange unknown values in time (j) to the left and known values (j + 1) to the right side:

$$\frac{\sigma_j^2 i^2-(q_{Sj}-\gamma_{Sj})^i}{4} V_{i-1,j}+ (- \frac{\sigma j^2 i^2}{2} - \frac{r_j}{2} - \frac{1}{\Delta t} V_{i,j}+ \frac{\sigma_j^2 i^2+(q_{Sj}-\gamma_{Sj})^i}{4} V_{i-1,j}$$

$$=- \frac{\sigma_j^2 i^2+(q_{Sj}-\gamma_{Sj})^i}{4} V_{i-1,j+1} -(- \frac{\sigma j^2 i^2}{2} - \frac{r_j}{2} - \frac{1}{\Delta t} V_{i-1,j} - \frac{\sigma_j^2 i^2+(q_{Sj}-\gamma_{Sj})^i}{4} V_{i-1,j+1},$$

$$(5.7)$$

and for simplicity we define a, b, c and d as:

$$a_{i,j} \equiv \frac{(\sigma_j^2\, i^2 - (q_{Sj} - \gamma_{Sj})^i}{4};$$

$$b_{i,j} \equiv \left(- \frac{\sigma_j^2\, i^2}{2} - \frac{r_j}{2} - \frac{1}{\Delta t} \right);$$

$$c_{i,j} \equiv \frac{(\sigma_j^2\, i^2 + (q_{Sj} - \gamma_{Sj})^i}{4};$$

$$d_{i,j} \equiv a_{i,j} V_{i-1,j+1} - \left(- \frac{\sigma_j^2\, i^2}{2} - \frac{r_j}{2} + \frac{1}{\Delta t} \right) V_{i,j+1} - c_{i,j} V_{i+1,j+1}. \tag{5.8}$$

The CN method gives us the following equation system for each j in matrix form:

$$\begin{bmatrix} b_{0,j} & c_{0,j} & 0 & 0 & . & . & . & . & 0 \\ a_{1,j} & b_{1,j} & c_{1,j} & 0 & . & . & . & . & . \\ 0 & a_{2,j} & b_{2,j} & c_{2,j} & 0 & . & . & . & . \\ . & 0 & a_{3,j} & b_{3,j} & c_{3,j} & 0 & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & a_{i,j} & b_{i,j} & c_{i,j} & 0 & . \\ . & . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & 0 & a_{n,j} & b_{n,j} \end{bmatrix} \begin{bmatrix} V_{0,j} \\ V_{1,j} \\ V_{2,j} \\ . \\ . \\ V_{i,j} \\ . \\ V_{n-1,j} \\ V_{n,j} \end{bmatrix} \begin{bmatrix} d_{0,j} \\ d_{1,j} \\ d_{2,j} \\ . \\ . \\ d_{i,j} \\ . \\ d_{n-1,j} \\ d_{n,j} \end{bmatrix}$$

To solve the system in the form PV = d for European options we use successive over-relaxation (SOR) algorithm. In comparison, from a computational point of view, SOR method offers a decent speed of convergence. Direct methods for tri-diagonal matrix are more efficient than indirect methods. If a tri-diagonal matrix is not the case, matrix inversion could be extremely time consuming and inefficient (Wilmott (2006)).

SOR indirect method solves equation systems iteratively. The solution will never be exact, but the accuracy is a user-defined parameter of the algorithm. The iterative solution process is known as the Jacobi iteration. From (5.9) we notice that the first equation can be written as:

$$a_{1,j} V_{0,j} + b_{1,j} V_{1,j} + c_{1,j} V_{2,j} = d_{1,j}, \tag{5.10}$$

so, generalizing this expression and rearranging terms we get:

$$V_{i,j} \equiv \frac{d_{i,j} - a_{i,j} V_{i-1,j} + c_{i,j} V_{i+1,j}}{b_{i,j}}, \tag{5.11}$$

The idea behind the Jacobi iteration is to make an initial guess for $V_{i,j}^{0} \equiv V_{i,j+1}$ (we will specify later boundary conditions in more detail but consider for now $V_{i,m-1}^{0} = V_{i,m}$ *for* $j = m-1$*)*, and iterations in k continue until the difference between $V_{i,j}^{k}$ and $V_{i,j}^{k+1}$ is sufficiently small for all $V_{i,j}$ at time step j (less or equal the error tolerance or desired accuracy):

$$V_{i,j}^{k+1} = \frac{d_{i,j} - a_{i,j} V_{i-1,j}^{k} + c_{i,j} V_{i+1,j}^{k}}{b_{i,j}} \tag{5.12}$$

Gauss-Seidel's improvement to the Jacobi method suggests using the most updated value as initial guess, which implies using $V_{i,j}^{k+1}$ immediately when available:

$$V_{i,j}^{k+1} = \frac{d_{i,j} - a_{i,j} V_{i-1,j}^{k+1} + c_{i,j} V_{i+1,j}^{k}}{b_{i,j}}, \tag{5.13}$$

SOR is another improvement that lays in the observation that $V_{i,j}^{k+1} = V_{i,j}^{k} + (V_{(i,j)}^{k+1} - V_{i,j}^{k})$, so the method over corrects faster the value of $V_{i,j}^{k}$, which is true if $V_{i,j}^{k}$ converge monotonically to $V_{i,j}$ in k. The SOR algorithm proposes (see Thomas (1999)):

$$y_{i,j}^{k+1} = \frac{d_{i,j} - a_{i,j} V_{i-1,j}^{k+1} + c_{i,j} V_{i+1,j}^{k}}{b_{i,j}},$$

$$V_{i,j}^{k+1} = V_{i,j}^{k} + \omega \, V_{i,j} - y_{i,j}^{k+1} \tag{5.14}$$

where $1 < \omega < 2$ is called the over-relaxation parameter. This parameter, which should lie between 1 and 2 (Thomas (1999)), speeds up the convergence to the true solution. The algorithm implemented varies the value of ω depending on the number of iterations taken to convergence. It takes an initial value of $\omega = 1$ and records the number of iterations over k required obtain the specified accuracy. In the next step j +1, if fewer iterations were needed, ω is increased by a small number (e.g. 0.05). While number of iterations continue decreasing we keep increasing the ω. If the number of iterations increase, we subtract a small number from ω. The intention is to choose ω to be the value that minimizes the number of iterations. If SOR matrix is time-homogeneous, then the over-relaxation parameter will remain unmodified. On the other hand, if there is a very strong time dependence in the matrix, the parameter will vary (see Wilmott (2006) and Smith (1985)).

(5.14)

We use the following boundary conditions for European options:

Call options

- $V_{0,j}=0, j=0,...,m$
- $V_{N,j}=N\Delta S\exp(-\int_{j\Delta t}^{T}\gamma_S(v)dv)-E\exp(-\int_{j\Delta t}^{T} r(v)dv), j=0,...,m-1$
- $V_{i,m})=(i\Delta S-E)^{+}, i=0,...,N-1$

Put options

- $V_{0,j}=E\exp(-\int_{j\Delta t}^{T} r(v)dv), j=0,...,m-1$
- $V_{N,j}=0, j=0,...,m$
- $V_{i,m})=(E-i\Delta S)^{+}\ i=0,...,N-1$

*E is the option's strike price

In the valuation of American options we have the free boundary condition $V(\tau,S(\tau))\geq H(S(\tau)),\ t\leq\tau\leq T$ (Duffy (2006)). In the CN finite-difference method context, this implies that every value of the option at the k+1 iteration is linked to every other value at every time step j and it is then necessary to modify the algorithm with an additional step, called projected SOR (PSOR) (see Cryer (1979)). This step can be used to solve other free-boundary PDEs for derivatives with more complex payoff functions (e.g. Bermudan options). The additional step for American options is simply substituting second expression in (5.14) for second expression in (5.15).

(5.14)

$$V_{i,j}^{k+1} = \frac{d_{i,j} - a_{i,j} V_{i-1,j}^{k+1} + c_{i,j} V_{i+1,j}^{k}}{b_{i,j}}$$

$$V_{i,j}^{k+1} = (V_{i,j}^{k} + \omega \; y_{i,j}^{k+1} - V_{i,j}^{k}) \vee H(i\Delta S))$$

We use the following boundary conditions for American options (Duffy (2006)):

Call options

- $V_{0,j}=0, j=0,\dots,m$
- $V_{N,j}=(N\Delta S-E)^{+}, j=0,\dots,m-1$
- $V_{i,m}=(i\Delta S-E)^{+}, i=0,\dots,N$

Put options

- $V_{0,j}=E, j=0,\dots,m$
- $V_{N,j}=0, j=0,\dots,m$
- $V_{i,m}=(E-i\Delta S)^{+} \; i=0,\dots,N-1$

5.1.2 CN scheme for PDE representation of derivative with CVA and FVA

If we recall (4.22) in subsection 4.2.1:

$$(\partial_t \hat{V} + A_t \hat{V} - (r+\lambda_B+\lambda_C)\hat{V} = -(R_B \lambda_B + \lambda_C)V^{-} - (\lambda_B + R_C \lambda_C)V^{+} + s_F V^{+}, \hat{V}(T,S)=H(S),) \qquad (5.16)$$

and linear PDE in (5.1), we can see that PDE in (5.16) is also a linear PDE which can be written in the form $\mathcal{L}\hat{V}=F(V)$, with source term F, that does not depend on $\hat{V}$. If we approximate partial derivatives as in 5.1.1, dropping the error terms, its finite-difference representation is:

$$\frac{V_{i,j+1} - V_{i,j}}{\Delta t} + \sigma_j^2 i_j^2 \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j} + V_{i+1,j+1} - 2V_{i,j+1} + V_{i-1,j+1}}{4} + (q_{Sj} - \gamma_{Sj}) i \frac{V_{i+1,j} - V_{i-1,j} + V_{i+1,j+1} - V_{i-1,j+1}}{4} - (r_j + \lambda_{Bj} + \lambda_{Cj}) \frac{V_{i,j} + V_{i,j+1}}{2}$$
$$= -(R_B \lambda_{Bj} + \lambda_{Cj}) \frac{V^{-}_{i,j} + V^{-}_{i,j+1}}{2} - (\lambda_{Bj} + R_C \lambda_{Cj}) \frac{V^{+}_{i,j} + V^{+}_{i,j+1}}{2} s_{Fj} \frac{V^{+}_{i,j} + V^{+}_{i,j+1}}{2} \qquad (5.17)$$

If we say $F_{i,j} \equiv -(R_B \lambda_{Bj} + \lambda_{Cj}) \dfrac{V^-_{i,j} + V^-_{i,j+1}}{2} - (\lambda_{Bj} + R_C \lambda_{Cj}) \dfrac{V^+_{i,j} + V^+_{i,j+1}}{2} s_{Fj} \dfrac{V^+_{i,j} + V^+_{i,j+1}}{2}$

and define a, ˆb, c and dˆas:

$$\begin{aligned}
a_{i,j} &= \frac{\sigma_j^2 i^2 - (q_{Sj} - \gamma_{Sj}) i}{4} \\
b_{i,j} &= (-\frac{\sigma_j^2 i^2}{2} - \frac{r_j + \lambda_{Bj} + \lambda_{Cj}}{2} - \frac{1}{\Delta t}) \\
c_{i,j} &= \frac{\sigma_j^2 i^2 - (q_{Sj} - \gamma_{Sj}) i}{4} \\
d^{o}_{i,j} &= a_{i,j} V_{i-1,j+1} - (-\frac{\sigma_j^2 i^2}{2} - \frac{r_j + \lambda_{Bj} + \lambda_{Cj}}{2} - \frac{1}{\Delta t}) V_{i,j+1} - c_{i,j} V_{i+1,j+1} + F_{i,j}
\end{aligned} \tag{5.18}$$

The CN scheme for V̂ can be written in matrix form for each j as:

$$\begin{bmatrix}
b_{0,j} & c_{0,j} & 0 & 0 & . & . & . & . & 0 \\
a_{1,j} & b_{1,j} & c_{1,j} & 0 & . & . & . & . & . \\
0 & a_{2,j} & b_{2,j} & c_{2,j} & 0 & . & . & . & . \\
. & 0 & a_{3,j} & b_{3,j} & c_{3,j} & 0 & . & . & . \\
. & . & . & . & . & . & . & . & . \\
. & . & . & . & a_{i,j} & b_{i,j} & c_{i,j} & 0 & . \\
. & . & . & . & . & . & . & . & . \\
0 & . & . & . & . & . & 0 & a_{n,j} & b_{n,j}
\end{bmatrix}
\begin{bmatrix}
V_{0,j} \\ V_{1,j} \\ V_{2,j} \\ . \\ . \\ V_{i,j} \\ . \\ V_{n-1,j} \\ V_{n,j}
\end{bmatrix}
\begin{bmatrix}
d_{0,j} \\ d_{1,j} \\ d_{2,j} \\ . \\ . \\ d_{i,j} \\ . \\ d_{n-1,j} \\ d_{n,j}
\end{bmatrix}$$

and it can be seen we have a problem in the same matrix form of 5.1.1, $\hat{P}\hat{V} = \hat{d}$, which can be solved using the same approach from previous subsection in the context of European and American options. Consider the following change in boundary conditions for European options with CVA and FVA:

Call options with CVA and FVA

- $\hat{V}_{0,j}=0,\ j=0,\ldots,m$
- $\hat{V}_{N,j}= \{N\Delta S\exp(-\int_{j\Delta t}^{T}\gamma_S(v)dv)-E\exp(-\int_{j\Delta t}^{T} r(v)dv)\}(1-(1-R_C)\int_{j\Delta t}^{T}\lambda_C(u)D_{r+\lambda\_B+\lambda\_C}(j\Delta t,u)du-\int_{j\Delta t}^{T} s_F(u)D_{r+\lambda\_B+\lambda\_C}(j\Delta t,u)du),\ j=0,\ldots,m-1$
- $\hat{V}_{i,m}=(i\Delta S-E)^{+},\ i=0,\ldots,N-1$

Put options with CVA and FVA

- $\hat{V}_{0,j}= \{E\exp(-\int_{j\Delta t}^{T} r(v)dv)\}(1-(1-R_C)\int_{j\Delta t}^{T}\lambda_C(u)D_{r+\lambda\_B+\lambda\_C}(j\Delta t,u)du-\int_{j\Delta t}^{T} s_F(u)D_{r+\lambda\_B+\lambda\_C}(j\Delta t,u)du),\ j=0,\ldots,m-1$
- $\hat{V}_{N,j})=0,\ j=0,\ldots,m$
- $\hat{V}_{i,m}=(E-i\Delta S)^{+}\ i=0,\ldots,N-1$

In the next subsection we provide a pseudo-code for the algorithm.

5.1.3 Pseudo-code for CN method

1. Compute boundary conditions according with region where the PDE is intended to be solved. In our case terminal, upper and lower boundaries for the value of the option in the mesh we have defined.

2. For j = m – 1,...,0

2a. Make initial guess for option values in j from known values in j + 1

2b. Compute upper boundaries for d if call option or lower boundaries if put option.

2c. For i = N – 1,...,1, compute values for remaining coefficients in matrix P. All are indexed in i (space) but will be overwritten at each time step j as coefficients are indexed in time in our solution. Matrix P depends on time as we assume our interest rates and volatility could be deterministic functions of time.

2d. Set number of loops equals zero.

2e. Loop until sum of squared errors is less an error tolerance.

2e1. Set sum of squared errors equals zero

2e2. For z = 1,...,N

2e2a. compute value of dz followed by $y_{z,j}^{k+1}$ and $V_{z,j}^{k+1}$ (remember (5.14) and (5.15)).

2e2b. Add squared error in z–th iteration ($V_{z,j}^{k+1}$ -$V_{z,}^{jk\ 2}$ to the sum of squared errors.

2e3. Add one iteration to the count of loops.

2f. If the count of in the j iteration is less than the count in the j + 1 iteration modify the parameter ω by a small number (see 5.1.1).

2g. Store the count of loops in the j iteration.

3. Return a matrix with the solution for the option price in defined time-space mesh.

5.2 European Option CVA and FVA with MC Simulation and Numerical Integration

For the CVA and FVA pricing of European options we propose to use the Euler-Maruyama method, which is one of most popular methods for single-asset price simulation (Wilmott (2006), Venegas (2008), Shreve (2004)).

From the risk neutral random walk for S:

$$dS(t) = (q_S(t) - \gamma_S(t))S(t)dt + \sigma(t)S(t)d\hat{W}(t), \quad (5.20)$$

$$0 \le t \le T$$

the following exact solution is obtained:

$$(S(T)=S(t)exp\{\int_t^T(q_S(u)-\gamma_S(u)-\frac{1}{2}\sigma^2(u))du+\int_t^T\sigma(u)d\,W(u)\},) \quad (5.21)$$

where W ˆ is a Wiener process under the risk neutral probability measure. The asset price is approximated through Euler-Maruyama method and Monte Carlo simulation (Wilmott (2006)), including Ito ˆ's stochastic integral regarding volatility as a deterministic function and a Wiener process (Venegas (2008)) in the following discrete representation:

$$S(T)=S(t)\ exp\left\{\int_t^T(q_S(u)-\gamma_S(u)-\frac{1}{2}\sigma^2(u))du\right\}\ exp\left\{\sum_{(j=1)}^{m}\sigma((j-1)\Delta t)\Delta W_j\right\}$$

$$d=S(t)\ exp\left\{\int_t^T(q_S(u)-\gamma_S(u)-\frac{1}{2}\sigma^2(u))du\right\}\ exp\left\{\sum_{(j=1)}^{m}\sigma((j-1)\Delta t)\sqrt{\Delta t}\,\theta_j\right\},$$

$$\theta_j\sim N(0,1) \quad (5.22)$$

The simulation of S(t) will converge to the exact solution as $m \to \infty\ \Delta t \to 0$. In our numerical approach, deterministic integral part of (5.22) is also computed by numerical integration so (5.22) can be written as:

$$(S(T)\approx S(t)\ exp\{\sum_{(j=1)}^{m}(q_S((j-1)\Delta t)-\gamma_S((j-1)\Delta t)-\frac{1}{2}\sigma^2((j-1)\Delta t))\Delta t\}$$

$$exp\{\sum_{(j=1)}^{m}\sigma((j-1)\Delta t)\sqrt{\Delta t}\,\theta_j\},@\theta_j\sim N(0,1))$$

$$\theta_j\sim N(0,1)$$

So, the value of an European derivative can be represented as an expected value under the risk-neutral probability measure as follows:

$$V(t,S(t))=E_t[H(S(T))]=D_k(t,T)E[H(S(T))], \quad (5.24)$$

and the value of an European derivative can be estimated following these steps (Wilmott et al. (1995) and Wilmott (2006)):

1. Simulate n risk-neutral random walks from solution (5.23) in m time steps with distance Δt until time T (expiry);

2. For each one of the i realizations of S(T) calculate derivative payoff $H(S(T)_i)$,i=1,…,n;

3. Calculate the average payoff;

4. and from the following observation:

$$\int_t^T D(t,u)_{r+\lambda\_B+\lambda\_C}\ \mathbb{E}_t[V^+(u,S(u))]du$$

$$=\int_t^T D(t,u)_r\, D(t,u)_{\lambda\_B+\lambda\_C}\ \mathbb{E}_t[V^+(u,S(u))]du$$

$$=\int_t^T D(t,u)^r\, D(t,u)_{\lambda\_B+\lambda\_C}\, D(u,T)_r\, E_t[V^+(T,S(T))]du$$

$$=\int_t^T D(t,u)_{\lambda\_B+\lambda\_C}\ D(t,T)_r\, \mathbb{E}_t[V^+(T,S(T))]du$$

$$=V^+(t,S(t))\int_t^T D(t,u)_{\lambda\_B+\lambda\_C}\ du,$$

$$(5.25)$$

It can be seen that values of $V^+(t,S(t))$ and $V^-(t,S(t))$are the present values $D(t,T)_r\ \mathbb{E}[V^+(T,S(T))]$ and $D(t,T)_r\ E[V^-(T,S(T))]$.

5. Then, after 1, 2 and 3 and observation in 4, U in (4.23) is computed by recurring to numerical integration methods for deterministic functions. We use R (2016) package developed by Piessens et al. (1983), which consists in adaptive quadrature of functions of one variable over a finite or infinite interval (See Appendix A).

5.3 CVA and FVA for American Options with MC Simulation, Least-Squares and Numerical Integration

In this section we propose an empirical approach for computation of CVA and FVA in Burgard and Kjaer (2011b)'s integral equation (4.23) in the context of American options. We take as starting point the method developed by Longstaff and Schwartz (2001) for American option's valuation, which is widely used among practitioners given its simple and intuitive implementation. The method consists in asset price simulation from time t to T and, as commonly known, the option holder evaluates at each point in time the benefit of exercising the option versus the expected value of continuation, exercising whether former is higher. The novelty of the method is that the conditional expected value of continuation is calculated using the cross-sectional information from the simulation and least-squares (Least-Squares Monte Carlo - LSM).

"The method consists in asset price simulation from time t to T and, as commonly known

The method uses a set of basis functions in the simulated asset prices. The fitted values are taken as the conditional value of continuation, later being compared with the immediate value of exercising. Moreno and Navas (2003) shows the robustness of the method, analyzing different sets of basis functions and its implications in the valuation of American derivatives. In the following sub-section, we present a summary of the original algorithm and the modification we propose to calculate CVA and FVA, given the convenient representation and computation of the expected conditional value throughout each iteration.

5.3.1 LSM Algorithm

We briefly introduce the algorithm, without extensive and rigorous description of definitions, proofs and consistency of the method. All these elements can be found in the original paper of Longstaff and Schwartz (2001).

It is assumed a probability space (Ω,F,P) and a finite temporal space [0,T]. The main interest is to determine the cash flows from American derivatives that take place in the defined temporal space. In particular, the value of American options is equivalent to the maximized value of discounted cash flows generated by the exercise of the option, where the maximum is taken over all stopping times with respect to the filtration F = FT . The path of cash flows generated by the option is denoted by $C(\omega,s;t,T)$, conditional on the absence of early exercise before time t and on the assumption that the option holder is following the optimal stopping strategy for all s, $t < s \leq T$.

LSM algorithm provide a path-wise approximation to the optimal stopping rule, maximizing option value. Although American options are continuously exercisable, it is assumed it can be exercised only in K times $0 < t1 < t2 < \ldots < tK < T$ to determine an optimal exercise policy. Cash flow from exercise at time tk is known by the investor, while value from continuation is not. The value of the option, assuming it cannot be exercised after time tk for any k, is the expectation of remaining discounted cash flows $C(\omega,s;t\_k,T)$ under the risk-free probability measure. The value of continuation is expressed as

$$G(\omega;t_k)=E^Q\left[\sum_{j=k+1}^{K} D_r(t_k,t_j)C(\omega,s;t_k,T)\,\middle|\,F_{tk}\right] \quad (5.26)$$

The LSM algorithm uses least-squares to approximate the value of function $G(\omega,.)$ at $tK-1,tK-2,\ldots,t1$. The algorithm works backward in time and if it is optimal to exercise the option at time tk+1, all previous values along path ω are set to zero. Because the functional form of $G(\omega,.)$ is unknown, it is set to be a linear combination of basis functions of a countable set of F_{tk} measurable basis functions on a function space (Longstaff and Schwartz (2001) and Moreno and Navas (2003)).

Once the subset of basis functions have been specified, the value of $G_B(\omega,t_{K-1})$ by regressing the discounted values of $C(\omega,s;t_{K-1},T)$ onto the basis functions for the paths where option is in the money at time tK−1. Only in-the-money paths are used. Fitted values are denoted by $\hat{G}B(\omega,t_{K-1})$. Then the stopping rule is given by

$$1_{\{\hat{G}_B(\omega,t_{K-1})<H(S(\omega,t_{K-1}))\}} \quad (5.27)$$

This exercise is repeated backwards in time for each path ω. At the end, the result is a matrix where all elements are either 1 or zero. As the stopping rule modifies all previous values of the matrix in the same path, the sum of all rows must be equals to 1. Now each 1 in the matrix should be substituted by the exercise value of the option at that point and discounted from the time of the optimal exercise to time t. The value of the option is given by the average of all present values at time t.

Pseudo-code

Define a matrix ANxM and store in it N paths for S, simulated with MC using (5.23), from t to T in K steps and a stopping strategy zero matrix Aˆ. Divide A by the strike price and use strike price as 1 when evaluating payoff function for normalization (Longstaff and Schwartz (2001));

Evaluate the payoff function in each position of A;

Discount each column *k = K, K −1, K −2,...,1* one step in time: $D(t_{k-1},t_k)_r$ A[.,k] and store discounted values in B;

In $\hat{k} = k - 1$, regress onto basis functions discounted in-the money values in step 3. in time tkˆ+1 against stock prices in each of the selected in-the-money paths but in time tkˆ . A linear combination of fitted values is $\hat{G}_B(\omega,t_k)$;

At time kˆ evaluate stopping rule (5.27) for $\hat{G}_B(\omega,t_{\hat{k}})$ and value of exercise at tkˆ.

Set to zero all previous values in A (as we are working backwards, that means future values), for each in-the-money path at k, where the stopping rule resulted in 1 and store stopping rule result in Aˆ;

repeat 4-6 from k = K – 1,...,1;

Compute $V = A\hat{A}'$ and discount all values $D(t,t_k)_r$ A[.,k], for *k=K,K-1,...,1;*

Take the average of discounted, greater than zero, values in 8.

5.3.2 Empirical approach for CVA and FVA for American options with LSM and Numerical Integration

In our empirical implementation we use just one basis function for simplicity. The code provided in Appendix A can be easily modified to include a set of basis functions.

The modification in our empirical approach is to take each column of V in 8. to calculate the average value of positive realizations as $\hat{\varepsilon}_t = E_t\,[V^+(t,S(t))]$ and the average value of negative realizations as $\hat{\eta}_t = E_t\,[V^-(t,S(t))]$).

Approximation for U in (4.23) is computed as follows:

$$U(t,S(t)) = -(1-R_B)\sum_{(j=1)}^{K}\lambda_B(t_j)\ \exp\{\sum_{(p=0)}^{j} r(t_p)+\lambda_B(t_p)+\lambda_C(t_p))\Delta t\}\ \eta_{tj}\ \Delta t$$

$$-(1-R_C)\sum_{(j=1)}^{K}\lambda_C(t_j)\ \exp\{\sum_{(p=0)}^{j} r(t_p)+\lambda_B(t_p)+\lambda_C(t_p))\Delta t\}\ \varepsilon_{tj}\ \Delta t$$

$$-\sum_{(j=1)}^{K} s_F(t_j)\ \exp\{\sum_{(p=0)}^{j} r(t_p)+\lambda_B(t_p)+\lambda_C(t_p))\Delta t\}\ \varepsilon_{tj}\ \Delta t$$

(5.28)

The results of this approach are shown in Chapter 6. Explicit code for this implementation can be found in Appendix A. As mentioned, this is an empirical approach.

06 | Results

In this chapter we show the results obtained from the implementation of solutions presented in Chapter 5.

Parameter	Value
E for Call	100
E for Put	100
r	0.05
r_B	0.08
r_C	0.10
λ_B	$\lambda_B 1 \; r_B - r$
λ_C	$\lambda_C 1 \; r_C - r$
s_F	$s_F 1 \; r_F - r$
R_B	0.4
R_C	0.4
σ	0.25
γ_S	0.07
q_S	0.06
S_{max}	300
T	5
m	500
N	500

Table 6.1: CN Solution to PDEs - Parameters

6.1 CN Solution to Black-Scholes PDE: European Options vs American Options

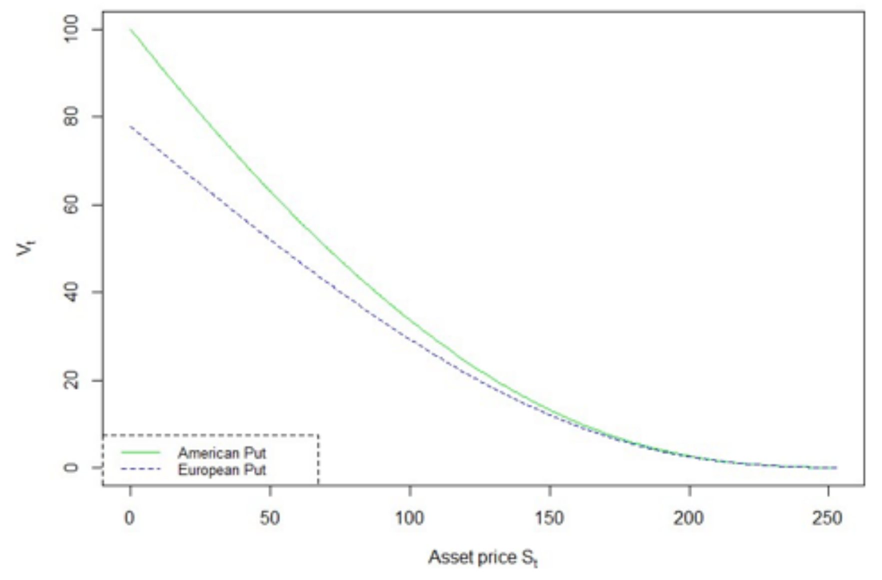


Figure 6.1: CN: American Put -vs- European Put

S_t	American Put	European Put
1 104.40	31.47	27.51
2 103.80	31.76	27.75
3 103.20	32.05	27.99
4 102.60	32.35	28.23
5 102.00	32.64	28.47
6 101.40	32.94	28.71
7 100.80	33.24	28.96
8 100.20	33.54	29.20
9 99.60	33.84	29.44
10 99.00	34.15	29.69
11 98.40	34.45	29.94
12 97.80	34.75	30.18
13 97.20	35.06	30.43
14 96.60	35.37	30.68
15 96.00	35.68	30.93
16 95.40	35.99	31.18

Table 6.2: CN: American Put -vs- European Put

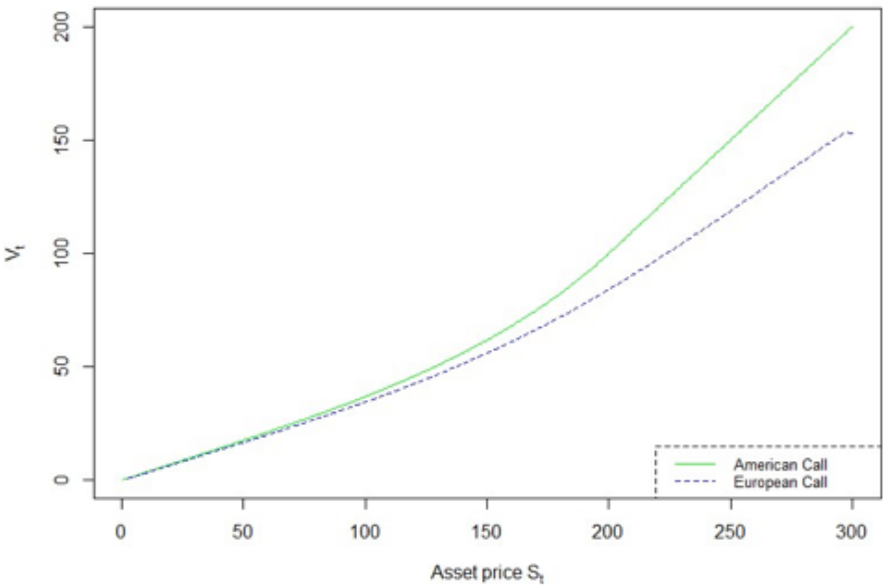


Figure 6.2:
CN: American Call -vs- European Call

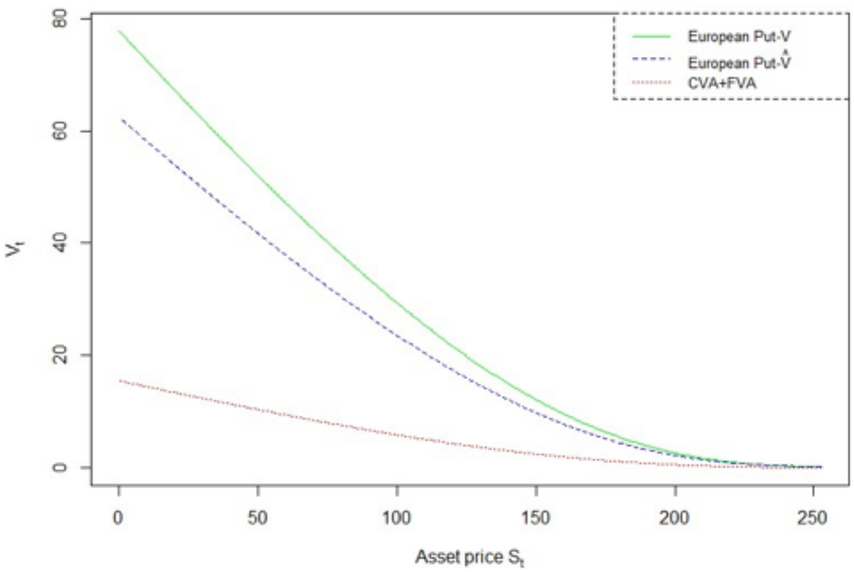
Table 6.3:
CN: American Call -vs- European Call

S_t	American Call	European Call
1 104.40	38.81	36.26
2 103.80	38.55	36.03
3 103.20	38.29	35.80
4 102.60	38.04	35.57
5 102.00	37.78	35.33
6 101.40	37.52	35.10
7 100.80	37.27	34.87
8 100.20	37.01	34.64
9 99.60	36.76	34.41
10 99.00	36.50	34.18
11 98.40	36.25	33.95
12 97.80	36.00	33.72
13 97.20	35.75	33.50
14 96.60	35.50	33.27
15 96.00	35.25	33.04
16 95.40	35.00	32.81

As shown in the charts above, our solutions to the risk-free Black-Scholes PDE reflect the principle that American option value should be always higher or equals European option value.

6.2 European Options: CN Solution to Black-Scholes PDE - vs - CN Solution Risky PDE

Figure 6.3: CN: European Put -vs- European Put with CVA and FVA



	S_t	V	$\hat{V}$	U
1	104.40	27.51	22.10	5.41
2	103.80	27.75	22.29	5.46
3	103.20	27.99	22.48	5.51
4	102.60	28.23	22.67	5.56
5	102.00	28.47	22.86	5.61
6	101.40	28.71	23.06	5.65
7	100.80	28.96	23.25	5.70
8	100.20	29.20	23.45	5.75
9	99.60	29.44	23.65	5.80
10	99.00	29.69	23.84	5.85
11	98.40	29.94	24.04	5.90
12	97.80	30.18	24.24	5.95
13	97.20	30.43	24.44	6.00
14	96.60	30.68	24.63	6.05
15	96.00	30.93	24.83	6.09
16	95.40	31.18	25.04	6.14

Table 6.4: CN: European Put -vs- European Put with CVA and FVA

Figure 6.4: CN: European Call -vs- European Call with CVA and FVA

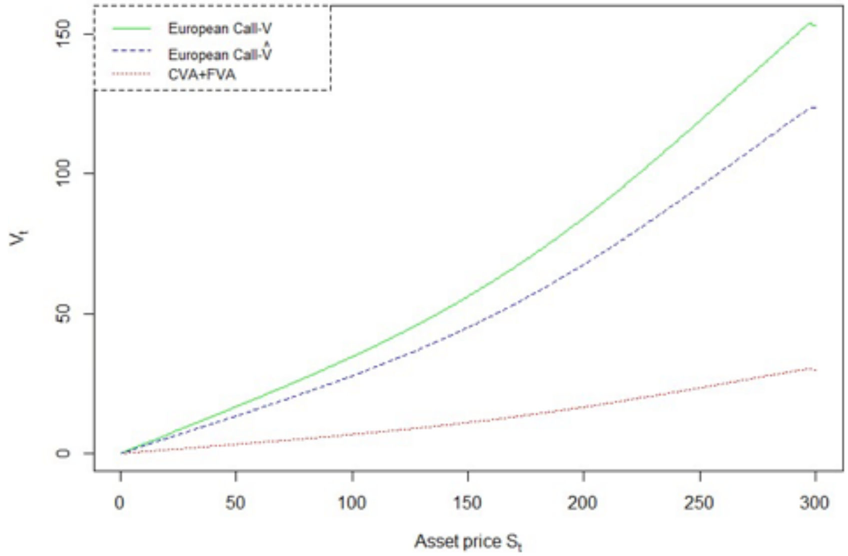


Table 6.5: CN: European Call -vs- European Call with CVA and FVA

	S_t	V	$\hat{V}$	U
1	104.40	36.26	29.11	7.15
2	103.80	36.03	28.93	7.10
3	103.20	35.80	28.74	7.06
4	102.60	35.57	28.55	7.01
5	102.00	35.33	28.37	6.97
6	101.40	35.10	28.18	6.92
7	100.80	34.87	28.00	6.87
8	100.20	34.64	27.81	6.83
9	99.60	34.41	27.63	6.78
10	99.00	34.18	27.44	6.74
11	98.40	33.95	27.26	6.69
12	97.80	33.72	27.07	6.65
13	97.20	33.50	26.89	6.60
14	96.60	33.27	26.71	6.56
15	96.00	33.04	26.53	6.51
16	95.40	32.81	26.35	6.47

European options are more exposed to counterparty risk as the only way out before maturity is due to an EoD or other termination event established by the parties in the MA. One could say with almost absolute certainty that in both latter cases the economic position of the surviving counterparty, in case it has a positive derivative value, in a default scenario is not as profitable as in a counterparty risk-free scenario.

6.3 American Options: CN Solution to Black-Scholes PDE - vs - CN Solution PDE with CVA and FVA

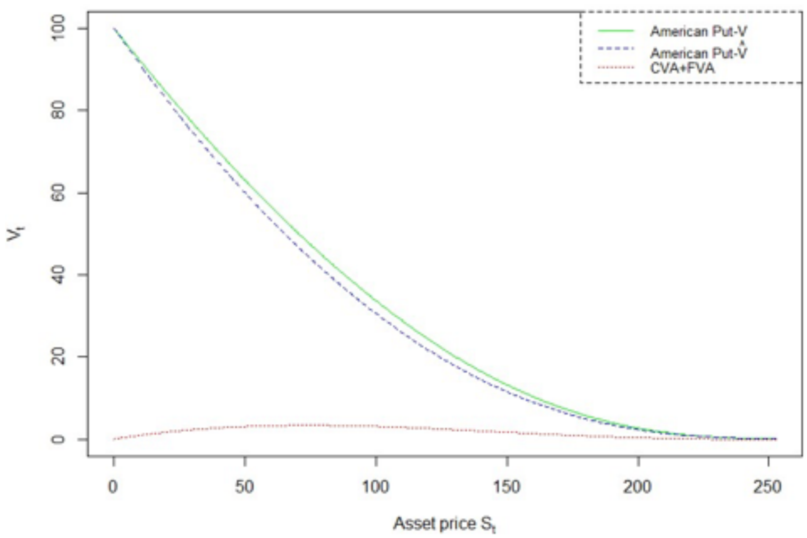


Figure 6.5: CN: American Put -vs- American Put with CVA and FVA

Table 6.6: CN: American Put -vs- American Put with CVA and FVA

	S_t	V	$\hat{V}$	U
1	104.40	31.47	28.46	3.01
2	103.80	31.76	28.74	3.02
3	103.20	32.05	29.02	3.03
4	102.60	32.35	29.30	3.05
5	102.00	32.64	29.58	3.06
6	101.40	32.94	29.87	3.07
7	100.80	33.24	30.16	3.08
8	100.20	33.54	30.44	3.10
9	99.60	33.84	30.73	3.11
10	99.00	34.15	31.03	3.12
11	98.40	34.45	31.32	3.13
12	97.80	34.75	31.61	3.14
13	97.20	35.06	31.91	3.15
14	96.60	35.37	32.21	3.16
15	96.00	35.68	32.50	3.17
16	95.40	35.99	32.81	3.18

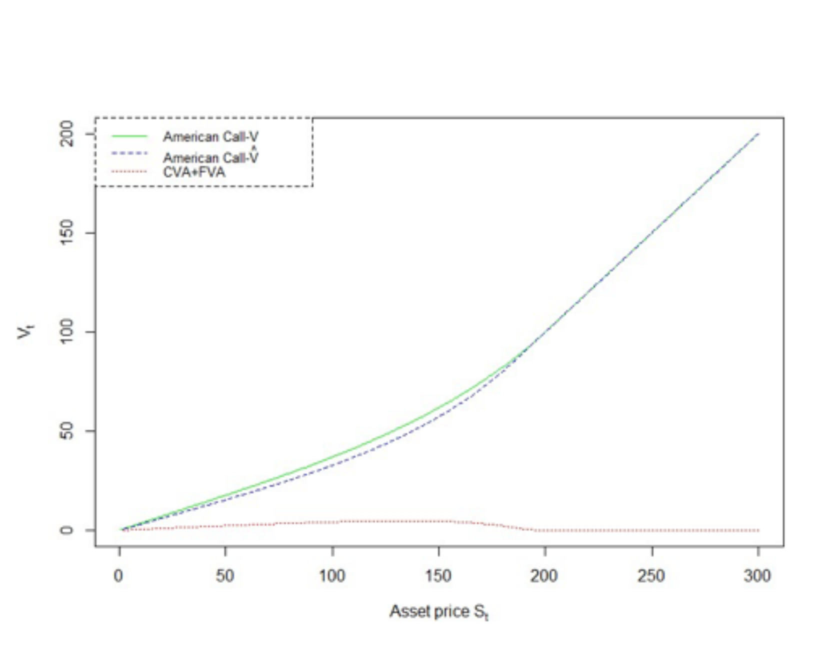


Table 6.5: CN: European Call -vs- European Call with CVA and FVA

	S_t	V	$\hat{V}$	U
1	104.40	38.81	34.51	4.31
2	103.80	38.55	34.26	4.29
3	103.20	38.29	34.02	4.27
4	102.60	38.04	33.78	4.26
5	102.00	37.78	33.54	4.24
6	101.40	37.52	33.30	4.22
7	100.80	37.27	33.06	4.21
8	100.20	37.01	32.82	4.19
9	99.60	36.76	32.59	4.17
10	99.00	36.50	32.35	4.15
11	98.40	36.25	32.12	4.13
12	97.80	36.00	31.88	4.12
13	97.20	35.75	31.65	4.10
14	96.60	35.50	31.42	4.08
15	96.00	35.25	31.19	4.06
16	95.40	35.00	30.96	4.04

Table 6.7: CN: American Call -vs- American Call with CVA and FVA

In the case of an American call option when the underlying asset price is close to the strike price it might not be optimal to early exercise the option but there is some counterparty risk and funding cost of due to the hedging strategy. When underlying asset price is high relative to the strike price, the adjustments are close to zero as one would exercise the option immediately. According with option valuation theory, it is never optimal to early exercise an American option on non-dividend paying stock (stock prices are supposed to drop down after dividend payments) if the option holder plans to maintain the stock in the future. In this case we have the effect of continuous dividends, CVA and FVA affecting the stopping rule criteria (adjusted continuation value against early exercise).

The CVA and FVA of American option is always less that the adjustments for European options. These makes sense since adjustments could be causing the early exercise, consistently with the possibility of early exercise due to counterparty risk reasons (counterparty's credit quality deterioration).

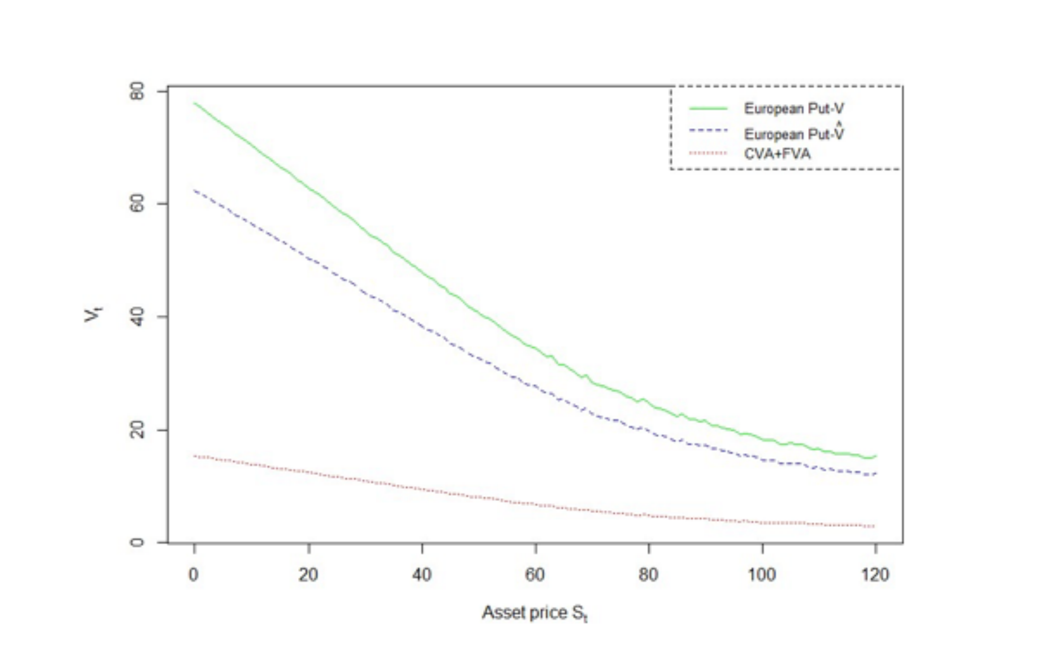


Figure 6.7: MC and NI: European Put -vs- European Put with CVA and FVA

	S_t	V	$\hat{V}$	U
1	90.00	21.56	17.30	4.27
2	91.00	21.65	17.37	4.28
3	92.00	20.73	16.63	4.10
4	93.00	20.80	16.68	4.11
5	94.00	20.36	16.33	4.03
6	95.00	20.14	16.16	3.98
7	96.00	19.92	15.98	3.94
8	97.00	19.20	15.40	3.80
9	98.00	19.33	15.50	3.82
10	99.00	19.17	15.37	3.79
11	100.00	18.88	15.14	3.73
12	101.00	18.27	14.66	3.61
13	102.00	18.28	14.67	3.62
14	103.00	18.32	14.69	3.62
15	104.00	17.51	14.04	3.46
16	105.00	17.55	14.08	3.47

Table 6.8: MC and NI: European Put -vs- European Put with CVA and FVA

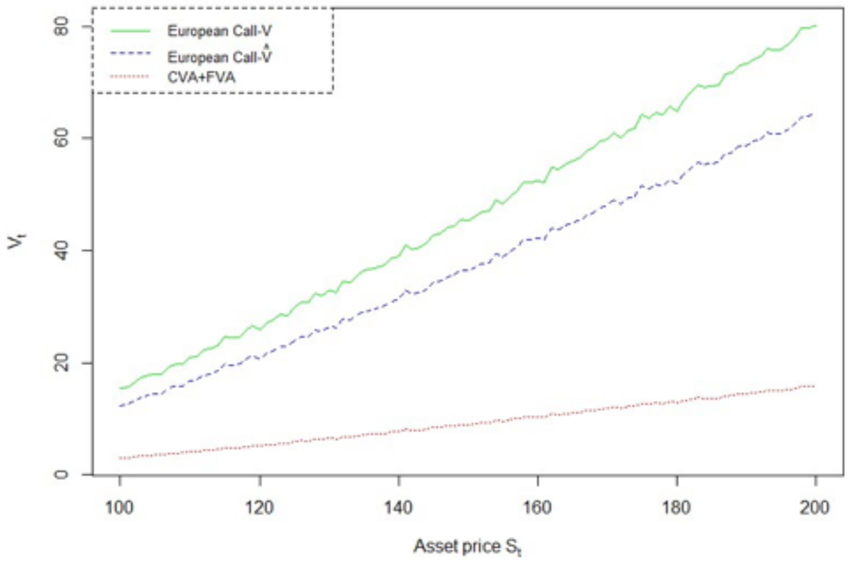


Table 6.9: MC and NI: European Call -vs- European Call with CVA and FVA

Table 6.9: MC and NI: European Call -vs- European Call with CVA and FVA

	S_t	V	$\hat{V}$	U
1	101.00	15.37	12.33	3.04
2	102.00	15.63	12.54	3.09
3	103.00	16.46	13.20	3.26
4	104.00	17.33	13.90	3.43
5	105.00	17.70	14.20	3.50
6	106.00	17.88	14.35	3.54
7	107.00	18.07	14.49	3.57
8	108.00	19.38	15.54	3.83
9	109.00	19.74	15.83	3.90
10	110.00	19.78	15.87	3.91
11	111.00	20.93	16.79	4.14
12	112.00	21.11	16.93	4.18
13	113.00	22.23	17.84	4.40
14	114.00	22.49	18.04	4.45
15	115.00	23.04	18.48	4.56
16	116.00	24.59	19.72	4.86

Adjustments to European option value computation through Monte Carlo and numerical integration show results with similar dynamic to those of the CN scheme solution. The difference in the presented option values is due to model calibration techniques, which are beyond the scope of this paper.

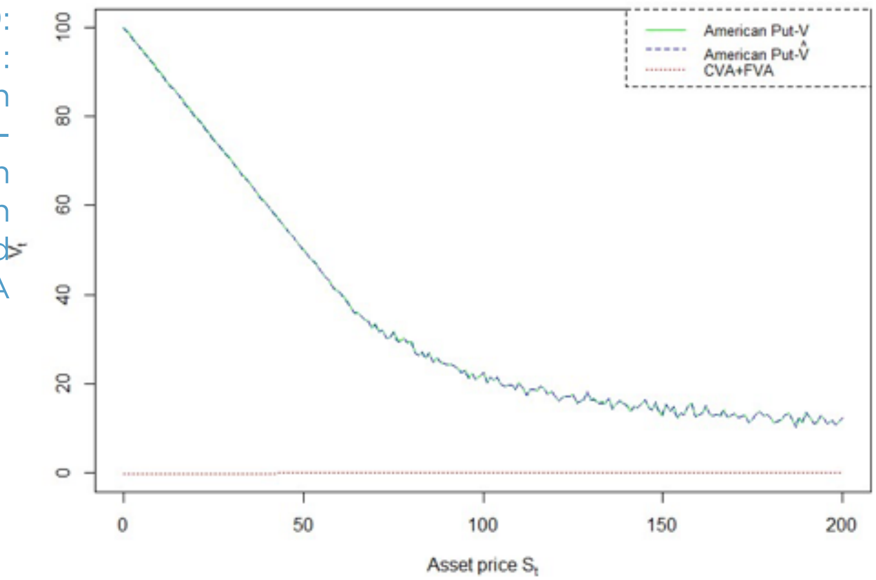


Figure 6.9: LSM and NI: American Put -vs- American Put with CVA and FVA

	S_t	V	$\hat{V}$	U
1	90.00	24.49	24.45	0.05
2	91.00	24.34	24.29	0.05
3	92.00	24.38	24.33	0.05
4	93.00	24.02	23.97	0.05
5	94.00	23.17	23.13	0.04
6	95.00	22.43	22.39	0.04
7	96.00	23.08	23.03	0.04
8	97.00	21.28	21.24	0.04
9	98.00	22.34	22.29	0.04
10	99.00	20.89	20.85	0.04
11	100.00	21.85	21.81	0.04
12	101.00	22.45	22.41	0.04
13	102.00	20.28	20.25	0.04
14	103.00	21.40	21.36	0.04
15	104.00	20.67	20.63	0.04
16	105.00	21.56	21.53	0.03

Table 6.10: LSM and NI: American Put -vs- American Put with CVA and FVA

Figure 6.10: LSM and NI: American Call -vs- American Call with CVA and FVA

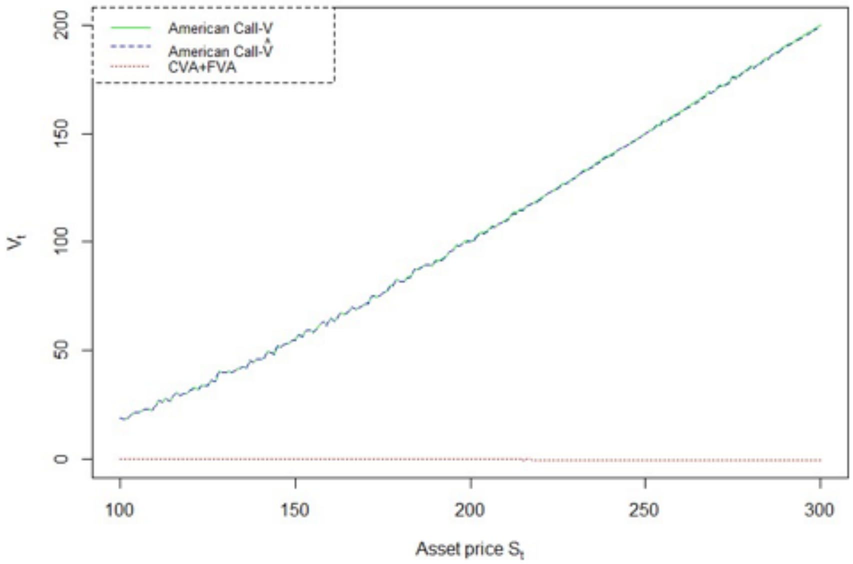


Table 6.11: LSM and NI: American Call -vs- American Call with CVA and FVA

	S_t	V	$\hat{V}$	U
1	100.00	18.96	18.92	0.03
2	101.00	18.49	18.45	0.04
3	102.00	18.98	18.95	0.04
4	103.00	20.55	20.51	0.04
5	104.00	21.55	21.51	0.04
6	105.00	21.51	21.46	0.04
7	106.00	22.15	22.10	0.04
8	107.00	22.87	22.82	0.04
9	108.00	23.18	23.13	0.05
10	109.00	22.62	22.57	0.05
11	110.00	24.35	24.30	0.05
12	111.00	27.18	27.13	0.05
13	112.00	26.17	26.11	0.05
14	113.00	27.95	27.90	0.05
15	114.00	26.65	26.59	0.05
16	115.00	29.09	29.03	0.06

Results for American option values through LSM are always very low or non-significant which could be explained by a computation error or the reason that in this modelling approach the American option value could be close to the strike price, always exercising the option making the adjustment value almost zero or dispersed in time where option adjustments are also non-significant. We think the problem is related to the definition of the stopping rule that considers the risk-free conditional expected value of the option instead the adjusted conditional expected value.

07 | Conclusions

In this document we presented numerical solutions to PDE representations for the value function of risk-free options and options with CVA and FVA through Crank-Nicolson finite-difference scheme, direct computation of CVA and FVA for European options through Monte Carlo simulation and numerical integration and proposed an empirical method for direct CVA and FVA computation through least-squares Monte Carlo and numerical integration for American options. These methods are well-known among practitioners and academics for pricing derivatives, mostly in the context of risk-free derivatives.

We found that finite-difference methods like CN could be more challenging to implement but the solutions they provide are computationally efficient and smoother when compared with Monte Carlo simulation results.

The results we found are consistent with option pricing theory for European and American options if we compare the functional forms in Chapter 6 with forms in most of the derivatives literature at the end of this document. Despite of this, accurate option values are beyond the scope of this paper and we did not deal with calibration techniques (e.g. defining properly upper bounds or a maximum price for the underlying asset, which may have a significant impact in the derivative's value).

Although we think the proposed empirical approach for American options with CVA and FVA through LSM and numerical integration might be consistent from a superficial mathematical perspective, more rigorous mathematical proofs and experiments are required to reach strong conclusions.

7.1 Future Extensions

» Explore in more depth calibration techniques for each of the methods presented here and compare derivatives' valuations and adjustments.

» Present solutions to PDEs for different derivatives e.g. forward contracts, interest rate swaps and exotic options.

» We also identified that boundary conditions for PDEs in the context of CVA and FVA valuation is a very relevant aspect that could have significant impact in final solutions and derivative values. Explore methods for determine boundary conditions or upper bounds for finite-difference adjusted valuation might result in better approximations. - Regarding the empirical approach using LSM and numerical integration, we identified that an improvement or solution to the proposed method might be to include an adjusted continuation value in the application of the stopping rule, which would also be consistent with our implementation for the CN risky PDE solution.

» Explore suggestions by Duffy (2004) regarding CN scheme or other finite-difference methods, testing stability and convergence.

» Extend the approach to a multi-asset portfolio with correlated assets.

» Extend the solutions and implementations for PDEs where interest rates and volatility are stochastic. In the case of volatility, a stochastic function of time and asset price.

7.1.1 Calibration for CVA and FVA in Emerging Markets

We consider additional promising development could be calibrating the model in the context of an emerging market country. OTC derivatives or portfolio of derivatives between two parties, where one or both parties may be based in an emerging market. To reach this calibration method we think it would be helpful to explore some cases before.

We have identified four cases for calibration of the presented CVA and FVA framework. For sure many more can be found. First case: a large financial institution A, the seller, and a company B, the buyer, where both parties have issued bonds in the same capital market or jurisdiction (also same currency), and bonds from A and B are sufficiently liquid to obtain a market yield. Second case: a large financial institution A, the seller, and a company C, the buyer, where only party A has issued bonds that are sufficiently liquid in the capital market. Party C may or may not have issued bonds. Cases 3 and 4 are the same in case 1 and 2 with the variation that counterparties B and C are now in an emerging market and obtain funding in a different currency and are exposed to a different country risk.

» First Case: Parties A and B have issued bonds that are liquid in the same market and denomination

In this case calibration of interest rates might be done straight forward from market yields.
» Second Case: Party A have issued bonds that are liquid

In this case calibration of interest rates might be done straight forward from market yields for party A and we would suggest that for party C one could define a peer group based on financial and credit metrics published by rating agencies where peers have bonds in the same market and currency and are liquid enough to build a benchmark market yield.

» Third and Fourth cases: Parties B and C are in an emerging market with a different currency

These cases sound like they could cover a large set of medium-large corporate entities of financial institutions in emerging markets that probably have credit ratings, they have issued bonds in their local market where they might be liquid enough. This case sounds more challenging as it would require developing a consistent method to incorporate FX risk and country risk in the calibration.

APPENDIX A

Explicit code in R (2016) for Solutions in Chapter 5

```
##----------Parameters/rates as defined in Burgard & Kjaer 2011-----------##
#install.packages("xtable") library(xtable)
r=function(t)0.05 #Risk-free rate
r_b=function(t)0.08 #Yield on recoveryless bond of seller B r_c=function(t)0.1
#Yield on recoveryless bond of counterparty C lambda_b=function(t)r_b(t)-r(t)
#Intensity of default seller B lambda_c=function(t)r_c(t)-r(t) #Intensity of default
counterparty C R_b=0.4 #Recovery rate on derivative value in case seller B defaults
R_c=0.4 #$Recovery rate on derivative value in case counterparty C defaults

#****r_F is the seller funding rate for borrowed cash on seller s derivatives replication
#cash account****
r_F=function(t)r(t) #if derivative can be used as collateral
r_F=function(t)r(t)+(1-R_b)*lambda_b(t) #if derivative cannot be used as collateral
s_F=function(t)r_F(t)-r(t)

##-----------Parameters for proposed solutions to PDEs--------------------##
#*In case coefficients are non-constant modify each parameter and specify
appropriate
#deterministic functions for each one
#*modify functions as well to make them time or space dependent

r_hat=function(t)r(t)+lambda_b(t)+lambda_c(t) g_s=function(t)0.07
#dividend yield q_s=function(t)0.06 #financing cost
sigma=function(t)0.25 #volatility T=5 #time to maturity in years
m=500 #number of time steps dt=T/(m) #time incr6ements
Smax=400 #max asset price N=500 #number of
space steps delta_s=Smax/(N) #space increments

##--------Crank-Nicolson Method - Risk Free - (American & European)--------##

crank_nicolson_bspde=function(smax=Smax,TtM=T,n_t=m,n_s=N,eps=1e-8,opt_
c=c("A","E"),
opt_t=c("C","P"),K=0){

deltas=smax/n_s
deltat=TtM/n_t omega=1.0
domega=0.05 oldloops=10000
s_v=c((n_s:1)*deltas,0)#zero is added as minimum price n_s=n_s+1
t_v=c(0,(1:n_t)*deltat) n_t=n_t+1 a=rep(0,n_s) b=rep(0,n_s) c=rep(0,n_s)
d=rep(0,n_s)
val=matrix(0,nrow=n_s,ncol=n_t)

#Boundary conditions if(opt_t=="C"){ ##Call
val[,n_t]=(s_v-K)*((s_v-K)>0)

if(opt_c=="E") for(p in 1:(n_t-1)){
val[1,p]=(smax*exp(-integrate(Vectorize(g_s),t_v[p],
t_v[n_t])$value)-K*exp(-integrate(Vectorize(r),t_v[p],t_v[n_t])$value))
}
```

```
if(opt_c=="A")for(p in 1:(n_t-1))val[1,p]=smax-K a[1] =0 b[1]=1 b[n_s]=1 c[n_s]=0
val[n_s,]=0
}

else{ ##Put
val[,n_t]=(K-s_v)*((K-s_v)>0)

if(opt_c=="E")for(l in 1:(n_t-1)){
val[n_s,l]=(K*exp(-integrate(Vectorize(r),t_v[l], t_v[n_t])$value))
}

if(opt_c=="A") val[n_s,]=K a[1] =0 b[1]=1 b[n_s]=1 c[n_s]=0 val[1,]=0
}

#Initial guess for V for(j in (n_t-1):1){
val[c(2:(n_s-1)),j]=val[c(2:(n_s-1)),j+1]

#Boundary conditions for d if(opt_t=="C"){ ##Call d[1]=val[1,j]
}
else{ ##Put
d[n_s]=val[n_s,j]
}

for(i in (n_s-1):2){
a[i]=(1/4*((sigma(t_v[j])^2)*i^2-(q_s(t_v[j])-g_s(t_v[j]))*i))                    b[i]=-
1/2*(sigma(t_v[j])^2)*i^2-r(t_v[j])/2-1/deltat c[i]=(1/4*((sigma(t_v[j])^2)*i^2+(q_s
(t_v[j])-g_s(t_v[j]))*i)) }

####SOR - Gauss-Seidel loops=0

repeat{
error=0
for(z in 2:(n_s-1)){
d[z]=(-(1/4*((sigma(t_v[j])^2)*z^2-(q_s(t_v[j])-g_s(t_v[j]))*z))*val[z-1,j+1]
-(-1/2*(sigma(t_v[j])^2)*z^2-r(t_v[j])/2+1/deltat)*val[z,j+1]
-(1/4*((sigma(t_v[j])^2)*z^2+(q_s(t_v[j])-g_s(t_v[j]))*z))*val[z+1,j+1])

y=(1/b[z])*(d[z]-a[z]*val[z-1,j]-c[z]*val[z+1,j])

if(opt_c=="A" & opt_t=="C"){#American Call

y=max(val[z,j]+omega*(y-val[z,j]),(s_v[z]-K)*((s_v[z]-K)>0))

}else if(opt_c=="A" & opt_t=="P" ){#American Put

y=max(val[z,j]+omega*(y-val[z,j]),(K-s_v[z])*((K-s_v[z])>0))

}else if(opt_c=="E"){#European option

y=val[z,j]+omega*(y-val[z,j])
```

```
}
error=error+(val[z,j]-y)^2 val[z,j]=y

} loops=loops+1
if(error<=eps)break
}

if(loops>oldloops)domega=-domega
omega=omega+domega oldloops=loops
}

return(cbind(s_v,val))
}
mydf=data.frame(crank_nicolson_bspde(opt_c = "E", opt_t = "C",K=110,n_t = 500,n_s = 500)) matplot(mydf[,1],mydf[,-1],type = "l")

#American call
amc=crank_nicolson_bspde(opt_c = "A", opt_t = "C",K=100)

#American put
amp=crank_nicolson_bspde(opt_c = "A", opt_t = "P",K=100)

#European call
euc=crank_nicolson_bspde(opt_c = "E", opt_t = "C",K=100)

#European put
eup=crank_nicolson_bspde(opt_c = "E", opt_t = "P",K=100)

#Risk-free American call vs European call

matplot(euc[which(euc[,2]>0.05),1],cbind(amc[which(euc[,2]>0.05),2], euc[which(euc[,2]>0.05),2]), type="l", pch=c(1,2), col = c("green", "blue"),
xlab =expression(paste("Asset price ",S[t])) , ylab = expression(V[t]))
legend("bottomright", legend=c("American Call", "European Call"),col=c("green", "blue"), lty=1:2, cex=0.8, box.lty=2)

# Table American call vs European call

amc_euc=data.frame(cbind(euc[which(95<euc[,1] & euc[,1]<105),1],amc[which(95<euc[,1] & euc[,1]<105),2],euc[which(95<euc[,1] & euc[,1]<105),2])) colnames(amc_euc)<-c("St","American Call", "European Call")

xtable(amc_euc)

# Table American put vs European Put

amp_eup=data.frame(cbind(eup[which(95<eup[,1] & eup[,1]<105),1],amp[which(95<eup[,1] & eup[,1]<105),2],eup[which(95<eup[,1] & eup[,1]<105),2])) colnames(amp_eup)<-c("St","American Put", "European Put")

xtable(amp_eup)
```

APPENDIX A

```
#-----------------------------Crank - Nicolson PDE Solution with CVA--------------###

crank_nicolson_bspde_CVA=function(val_reg=valr,smax=Smax,TtM=T,n_t=m,n_
s=N,eps=1e-8, opt_c=c("A","E"),opt_t=c("C","P"),K=0){

deltas=smax/n_s deltat=TtM/n_t
omega=1.0 domega=0.05 oldloops=10000
s_v=c((n_s:1)*deltas,0) n_s=n_s+1
t_v=c(0,(1:n_t)*deltat) n_t=n_t+1
a=rep(0,n_s) b=rep(0,n_s)
c=rep(0,n_s) d=rep(0,n_s)
val=matrix(0,nrow=n_s,ncol=n_t)

#Boundary conditions if(opt_t=="C"){ ##Call

val[,n_t]=(s_v-K)*((s_v-K)>0)

if(opt_c=="E") for(p in 1:(n_t-1)){

v1=(smax*exp(-integrate(Vectorize(g_s),t_v[p],t_v[n_t])$value) -K*exp(-integrate(V
ectorize(r),t_v[p],t_v[n_t])$value))

cva=(-(1-R_b)*integrate(Vectorize(function(t)lambda_b(t)*
exp(-(integrate(Vectorize(lambda_b),t_v[p],t)$value+integrate(Vectorize(lambda
_c)
,t_v[p],t)$value))*min(v1,0)),t_v[p],t_v[n_t])$value-(1-R_c)
*integrate(Vectorize(function(t)lambda_c(t)*exp(-(integrate(Vectorize(lambda_b),
t_v[p],t)$value+integrate(Vectorize(lambda_c),t_v[p],t)$value))*max(v1,0)),t_v[p],
t_v[n_t])$value-integrate(Vectorize(function(t)s_F(t)*exp(-(integrate(Vectorize
(lambda_b),t_v[p],t)$value+integrate(Vectorize(lambda_c),t_v[p],t)$value))
*max(v1,0)),t_v[p],t_v[n_t])$value)

val[1,p]=v1+cva

}
if(opt_c=="A")for(p in 1:(n_t-1))val[1,p]=smax-K

a[1] =0 b[1]=1
b[n_s]=1 c[n_s]=0
d[n_s]=0
val[n_s,]=0

}
else{ ##Put
val[,n_t]=(K-s_v)*((K-s_v)>0)

if(opt_c=="E")for(l in 1:(n_t-1)){
v2=(K*exp(-integrate(Vectorize(r),t_v[l],t_v[n_t])$value))

cva=(-(1-R_b)*integrate(Vectorize(functon(t)lambda_b(t)*exp(-(integrate(Vectorize
```

```
(lambda_b),t_v[l],t)$value+integrate(Vectorize(lambda_c),t_v[l],t)$value))*min(v2,0))
,t_v[l],t_v[n_t])$value-(1-R_c)*integrate(Vectorize(function(t)lambda_c(t)
   *exp(-(integrate(Vectorize(lambda_b),t_v[l],t)$value+integrate(Vectorize(lambda
_c)
   ,t_v[l],t)$value))*max(v2,0)),t_v[l],t_v[n_t])$value-integrate(Vectorize(function(t)
s_F(t)*exp(-(integrate(Vectorize(lambda_b),t_v[l],t)$value+integrate(Vectorize(lamb
da_c),t_v[l],t)$value))*max(v2,0)),t_v[l],t_v[n_t])$value)

   val[n_s,l]= v2+cva
   }
   if(opt_c=="A") val[n_s,]=K
   a[1] =0
   b[1]=1 b[n_s]=1 c[n_s]=0
   d[1]=0 val[1,]=0

   }
   #Initial guess for V for(j in (n_t-1):1){
   val[c(2:(n_s-1)),j]=val[c(2:(n_s-1)),j+1]

   #Boundary conditions for d if(opt_t=="C"){ ##Call
   d[1]=(val[1,j]-(R_b*lambda_b(t_v[j])+lambda_c(t_v[j]))*min((val_reg[1,j]+val_
reg[1,j+1])/2
   -(R_c*lambda_c(t_v[j])+lambda_b(t_v[j]))*max((val_reg[1,j]+val_
reg[1,j+1])/2,0)+s_F(t_v[j]
   *max((val_reg[1,j]+val_reg[1,j+1])/2,0))
   }

   else{ ##Put
   d[n_s]=(val[n_s,j]-(R_b*lambda_b(t_v[j])+lambda_c(t_v[j]))*min((val_reg[1,j]+val_
reg[1,j+1
   -(R_c*lambda_c(t_v[j])                    +lambda_b(t_v[j]))*max((val_reg[1,j]+val_
reg[1,j+1])/2,0)+s_F(t_v[j
   *max((val_reg[1,j]+val_reg[1,j+1])/2,0))

   }
   for(i in (n_s-1):2){ a[i]=(1/4*((sigma(t_v[j])^2)*i^2-(q_s(t_v[j])-g_s(t_v[j]))*i))
   #r+lambda_b+lambda_c
   b[i]=-1/2*(sigma(t_v[j])^2)*i^2-r_hat(t_v[j])/2-1/deltat c[i]=(1/4*((sigma(t_v[j])^
2)*i^2+(q_s(t_v[j])-g_s(t_v[j]))*i))
   }

   ####SOR - Gauss-Seidel loops=0
   repeat{
   error=0 for(z in 2:(n_s-1)){
   d[z]=(-(1/4*((sigma(t_v[j])^2)*z^2-(q_s(t_v[j])-g_s(t_v[j]))*z))
   *val[z-1,j+1]-(-1/2*(sigma(t_v[j])^2)*z^2-r_hat(t_v[j])/2+1/deltat)
   *val[z,j+1]-(1/4*((sigma(t_v[j])^2)*z^2+(q_s(t_v[j])-g_s(t_v[j]))*z))
   *val[z+1,j+1]-(R_b*lambda_b(t_v[j])+lambda_c(t_v[j]))*min((val_reg[z,j]+val_reg[z,j
-(R_c*lambda_c(t_v[j])+lambda_b(t_v[j]))*max((val_reg[z,j]+val_reg[z,j+1])/2,0) +s_
   *max((val_reg[z,j]+val_reg[z,j+1])/2,0))
```

```
y=(1/b[z])*(d[z]-a[z]*val[z-1,j]-c[z]*val[z+1,j])

if(opt_c=="A" & opt_t=="C"){#American Call y=max(val[z,j]+omega*(y-val[z,j]),(s_v[z]-K)*(s_v[z]>K))

}else if(opt_c=="A" & opt_t=="P" ){#American Put

y=max(val[z,j]+omega*(y-val[z,j]),(K-s_v[z])*(K>s_v[z]))

}else if(opt_c=="E"){#European option

y=val[z,j]+omega*(y-val[z,j])
}

error=error+(val[z,j]-y)^2 val[z,j]=y
} loops=loops+1 if(error<eps)break
}

if(loops>oldloops)domega=-domega
omega=omega+domega oldloops=loops
}

return(cbind(s_v,val))
}

##American call with CVA & FVA valr=amc
amc_cva=crank_nicolson_bspde_CVA(val_reg=valr,opt_c = "A", opt_t = "C",K=100)

matplot(amc[which(amc[,2]>0.05),1],cbind(amc[which(amc[,2]>0.05),2],
amc_cva[which(amc[,2]>0.05),2],amc[which(amc[,2]>0.05),2]
-amc_cva[which(amc[,2]>0.05),2]), type="l", pch=c(1,2,3), col = c("green", "blue", "red"), xlab =expression(paste("Asset price ",S[t])) , ylab = expression(V[t]))

legend("topleft", legend=c(expression(paste("American Call-",V)),
expression(paste("American Call-",hat(V))),"CVA+FVA"),col=c("green", "blue", "red"), lty=1:3, cex=0.8, box.lty=2)

# Table American call vs American call CVA & FVA
amc_amccva=data.frame(cbind(amc[which(95<amc[,1] & amc[,1]<105),1]
,amc[which(95<amc[,1] & amc[,1]<105),2],amc_cva[which(95<amc[,1] & amc[,1]<105),2], amc[which(95<amc[,1] & amc[,1]<105),2]-amc_cva[which(95<amc[,1] & amc[,1]<105),2])) colnames(amc_amccva)<-c("St","American Call-V", "American Call-(V)","U")

xtable(amc_amccva)

##American put with CVA & FVA valr=amp amp_cva=crank_nicolson_bspde_CVA(val_reg=valr,opt_c = "A", opt_t = "P",K=100)

matplot(amp[which(amp[,2]>0.05),1],cbind(amp[which(amp[,2]>0.05),2],
amp_cva[which(amp[,2]>0.05),2],amp[which(amp[,2]>0.05),2]
```

```
-amp_cva[which(amp[,2]>0.05),2]), type="l", pch=c(1,2,3), col = c("green",
"blue", "red"), xlab =expression(paste("Asset price ",S[t])) , ylab = expression(V[t]))
legend("topright", legend=c(expression(paste("American Put-",V)),
expression(paste("American Put-",hat(V))),"CVA+FVA"),col=c("green", "blue", "red"),
lty=1:3, cex=0.8, box.lty=2)

# Table American put vs American put CVA & FVA
amp_ampcva=data.frame(cbind(amp[which(95<amp[,1] & amp[,1]<105),1]
,amp[which(95<amp[,1] & amp[,1]<105),2],amp_cva[which(95<amp[,1]
& amp[,1]<105),2], amp[which(95<amp[,1] & amp[,1]<105),2]-amp_
cva[which(95<amp[,1] & amp[,1]<105),2]))
colnames(amp_ampcva)<-c("St","American Put-V", "American Put-(V)","U")
xtable(amp_ampcva)

##European call with CVA & FVA valr=euc
euc_cva=crank_nicolson_bspde_CVA(val_reg=valr,opt_c = "E", opt_t = "C",K=100)
matplot(euc[which(euc[,2]>0.05),1],cbind(euc[which(euc[,2]>0.05),2],
euc_cva[which(euc[,2]>0.05),2],euc[which(euc[,2]>0.05),2]
-euc_cva[which(euc[,2]>0.05),2]), type="l", pch=c(1,2,3), col = c("green", "blue",
"red"), xlab =expression(paste("Asset price ",S[t])) , ylab = expression(V[t]))
legend("topleft", legend=c(expression(paste("European Call-",V)),
expression(paste("European Call-",hat(V))),"CVA+FVA"),col=c("green", "blue",
"red"), lty=1:3, cex=0.8, box.lty=2)

# Table European call vs European call CVA & FVA
euc_euccva=data.frame(cbind(euc[which(95<euc[,1] & euc[,1]<105),1]
,euc[which(95<euc[,1] & euc[,1]<105),2],euc_cva[which(95<euc[,1] &
euc[,1]<105),2], euc[which(95<euc[,1] & euc[,1]<105),2]-euc_cva[which(95<euc[,1]
& euc[,1]<105),2])) colnames(euc_euccva)<-c("St","V", "(V)","U")
xtable(euc_euccva)

##European put with CVA & FVA valr=eup
eup_cva=crank_nicolson_bspde_CVA(val_reg=valr,opt_c = "E", opt_t = "P",K=100)
matplot(eup[which(eup[,2]>0.05),1],cbind(eup[which(eup[,2]>0.05),2], eup_
cva[which(eup[,2]>0.05),2],eup[which(eup[,2]>0.05),2]
-eup_cva[which(eup[,2]>0.05),2]), type="l", pch=c(1,2,3), col = c("green", "blue",
"red"), xlab =expression(paste("Asset price ",S[t])) , ylab = expression(V[t]))
legend("topright", legend=c(expression(paste("European Put-",V)),
expression(paste("European Put-",hat(V))),"CVA+FVA"),col=c("green", "blue",
"red"), lty=1:3, cex=0.8, box.lty=2)

# Table European put vs European put CVA & FVA eup_eupcva=data.
frame(cbind(eup[which(95<eup[,1] & eup[,1]<105),1]
,eup[which(95<eup[,1] & eup[,1]<105),2],eup_cva[which(95<eup[,1] &
eup[,1]<105),2], eup[which(95<eup[,1] & eup[,1]<105),2] -eup_cva[which(95<eup[,1]
& eup[,1]<105),2]))

colnames(eup_eupcva)<-c("St","V", "{V}","U")

xtable(eup_eupcva)
```

```
##---------------------------------Monte Carlo Simulation-----------------------------###
S0=100 npaths=1000 m=1000

path_mat=matrix(S0,nrow=m+1,ncol=npaths)
path_mat[c(2:(m+1)),]=(t(sapply((1:m)*dt,r)*S0*dt+t(matrix(rnorm(npaths*m),
nrow=m,ncol=npaths))%*%diag(sapply((1:m)*dt,sigma)*S0*sqrt(dt)))) path_
mat=apply(path_mat,2,cumsum) matplot((0:m)*dt,path_mat,type="l")

#-- European Option CVA with MC Simulation and Numerical Integration - Main
Result 3-------

europeanOpts_CVA_
MC=function(opt=c("P","C"),St=100,npaths=1000,m=1000,E=100,Tmin=0,Tmax=5){
dt=(Tmax-Tmin)/m

#Simulation of n asset prices ST=Ste{det+sto} path_
mat=matrix(St,nrow=m+1,ncol=npaths)
path_mat[c(2:(m+1)),]=(t(sapply((1:m)*dt,function(t)q_s(t)-g_s(t))*St*dt+t(matrix
(rnorm(npaths*m),nrow=m,ncol=npaths))%*%diag(sapply((1:m)*dt,sigma)*St*sqrt
(dt))))
path_mat=apply(path_mat,2,cumsum)
S_T=path_mat[m+1,]
S_T[which(S_T<0)]=0 #Truncate stock prices at 0 #Compute option expected value
in Tmax E[V(T,S(T))]
payoff=(S_T-E)*((S_T-E)>0)*(opt=="C")+(E-S_T)*(0<(E-S_T))*(opt=="P") exp_
val=mean(payoff)
#value of E[V(t,S(T))|Ft]=Dr(t,T)E[V(t,S(T))]
V0=exp(-integrate(Vectorize(r),Tmin,Tmax)$value)*exp_val
V_neg_exp_pv=exp(-integrate(Vectorize(r),Tmin,Tmax)$value)*mean(sapply(payof
f,min,0)) V_pos_exp_pv=exp(-integrate(Vectorize(r),Tmin,Tmax)$value)*mean(sapply
(payoff,max,0))
CVA=(-(1-R_b)*V_neg_exp_pv*integrate(Vectorize(function(t)lambda_b(t)
*exp(-(integrate(Vectorize(lambda_b),
Tmin,t)$value+integrate(Vectorize(lambda_c),Tmin,t)$value))),Tmin,Tmax)$value-
(1-R_c)
*V_pos_exp_pv
*integrate(Vectorize(function(t)lambda_c(t)*exp(-(integrate(Vectorize(lambda_b),
Tmin,t) $value+integrate
(Vectorize(lambda_c),Tmin,t)$value))),Tmin,Tmax)$value-V_pos_exp_pv
*integrate(Vectorize(function(t)s_F(t)
*exp(-(integrate(Vectorize(lambda_b),Tmin,t)$value+integrate(Vectorize(lambda
_c), Tmin,t)$value))),
Tmin,Tmax)$value)
return(c(V0,CVA))
}

europeanOpts_CVA_MC(opt = "C",St=100,E=100)
##European call with CVA & FVA eup[which(95<eup[,1] & eup[,1]<105),1]

vect_eurc_cva=sapply(c(100:200),europeanOpts_CVA_MC,opt =
"C",npaths=10000,m=1000,
```

```
E=100,Tmin=0,Tmax=5) vect_eurc_cva[2,]=-vect_eurc_cva[2,]
matplot(c(100:200),cbind(t(vect_eurc_cva)[,1],t(vect_eurc_cva)[,1]-t(vect_eurc_
cva)[,2], t(vect_eurc_cva)[,2]), type="l",pch=c(1,2,3), col = c("green", "blue", "red"),
xlab =expression(paste("Asset price ",S[t])) , ylab = expression(V[t])) legend("topleft",
legend=c(expression(paste("European Call-",V)),
expression(paste("European Call-",hat(V))),"CVA+FVA"),col=c("green", "blue",
"red"), lty=1:3, cex=0.8, box.lty=2)

# Table European call vs European call CVA & FVA
mc_euccva=data.frame(cbind(c(101:116),t(vect_eurc_cva)[c(1:16),1],
t(vect_eurc_cva)[c(1:16),1]-t(vect_eurc_cva)[c(1:16),2],t(vect_eurc_cva)
[c(1:16),2])) colnames(mc_euccva)<-c("St","V", "(V)","U")
xtable(mc_euccva)

##European put with CVA & FVA
vect_eurp_cva=sapply(c(0:120),europeanOpts_CVA_MC,opt = "P", npaths=10000,
m=1000,E=100,Tmin=0,Tmax=5)
vect_eurp_cva[2,]=-vect_eurp_cva[2,]
matplot(c(0:120),cbind(t(vect_eurp_cva)[,1],t(vect_eurp_cva)[,1]-t(vect_eurp_cva)
[,2], t(vect_eurp_cva)[,2]), type="l",pch=c(1,2,3), col = c("green", "blue", "red"), xlab
=expression(paste("Asset price ",S[t])) , ylab = expression(V[t])) legend("topright",
legend=c(expression(paste("European Put-",V)),
expression(paste("European Put-",hat(V))),"CVA+FVA"),col=c("green", "blue",
"red"), lty=1:3, cex=0.8, box.lty=2)

# Table European put vs European put CVA & FVA
mc_eupcva=data.frame(cbind(c(90:105),t(vect_eurp_cva)[c(90:105),1],
t(vect_eurp_cva)[c(90:105),1]-t(vect_eurp_cva)[c(90:105),2], t(vect_eurp_cva)
[c(90:105),2])) colnames(mc_eupcva)<-c("St","V", "(V)","U")
xtable(mc_eupcva)

#--------American Option CVA with MC Simulation, Least-Squares and Numerical

#Integration - Main Result 3---------------------------------------------------

americanOpts_CVA_
LSM=function(opt=c("P","C"),St=100,npaths=1000,m=100,E=100,Tmin=0,
Tmax=5,rb=R_b,rc=R_c){
dt=(Tmax-Tmin)/m
path_mat=matrix(St,nrow=m+1,ncol=npaths)
path_mat[c(2:(m+1)),]=(t((sapply((1:m)*dt,function(t)
q_s(t)-g_s(t)))*St*dt+t(matrix
(npaths*m),nrow=m,ncol=npaths))%*%diag(sapply((1:m)*dt,sigma)*St*sqrt(dt))))
path_mat=apply(path_mat,2,cumsum)
for(i in 1:npaths)if(min(path_mat[,i])<0)path_mat[c(which(path_mat[,i]<0)
[1]:m),i]=0

#if a path touches

#0, all values after are set to zero
val_mat=(E-path_mat)*(opt=="P")+(path_mat-E)*(opt=="C") val_mat=(val_
```

```
mat*(val_mat>0))/E path_mat=path_mat/E
   st_mat=matrix(1,nrow=m+1,ncol=npaths) st_mat[m+1,which(val_
mat[m+1,]==0)]=0
   for(i in m:1){
   disc_rf=(mapply(function(a,b){exp(-integrate(Vectorize(r),a,b)$value)},
a=rep(i*dt,m-i+1),b=c(i:m)*dt))
   if(i==m){
   pos=which(val_mat[(i+1):(m+1),]>0)
   model=(lm(y~x1+x2,data = data.frame(cbind(y=val_mat[(i+1):(m+1),pos]*disc_rf,
   x1=path_mat[i,pos],x2=path_mat[i,pos]^2)),na.action=na.omit))
   }else{
   pos=which(apply(val_mat[(i+1):(m+1),],2,max,0)>0)
   model=(lm(y~x1+x2,data = data.frame(cbind(y=apply(t(val_mat[(i+1):(m+1),pos])
   %*%diag(disc_rf),1,max),x1=path_mat[i,pos],x2=path_mat[i,pos]^2)),na.action=na.
omit))
   }
   st_mat[i,pos]=(predict(model)<val_mat[i,pos])*1 st_mat[c((i+1):(m+1)),which(st_
mat[i,]>0)]=0 val_mat=st_mat*val_mat
   }
   disc_rf=mapply(function(a,b){exp(-integrate(Vectorize(r),a,b)$value)}
   ,a=rep(Tmin,m+1),b=c(0:m)*dt)

   ##Longstaff & Schwartz value
   opt_val=mean(apply(t(val_mat*E)%*%diag(disc_rf),1,max))

   ###Alternative Longstaff & Schwartz value opt_val2= sum(apply(t(val_
mat*E),2,mean)*disc_rf)

   # expected value of positive derivative values (Expected positive exposure) opt_
exp_val_pos=apply(t(val_mat*E)*(t(val_mat*E)>0),2,mean)

   #expected value of negative derivative values (Expected negative exposure) opt_
exp_val_neg=apply(t(val_mat*E)*(t(val_mat*E)<0),2,mean)
   CVA=0 for(j in 1:m){
   CVA=(CVA-(1-rb)*integrate(Vectorize(function(t)lambda_b(t) *exp(-
integrate(Vectorize(r_hat)
   ,Tmin,t)$value)*opt_exp_val_neg[j]),(j-1)*dt,j*dt)$value
   -(1-rc)*integrate(Vectorize(function(t)lambda_c(t)*exp(-integrate(Vectorize(r_
hat),
   Tmin,t)$value)*opt_exp_val_pos[j]),(j-1)*dt,j*dt)$value
   -integrate(Vectorize(function(t)s_F(t)*exp(-integrate(Vectorize(r_
hat),Tmin,t)$value)
   *opt_exp_val_pos[j]),(j-1)*dt,j*dt)$value)
   } return(c(opt_val,opt_val+CVA,CVA))
   }
   americanOpts_CVA_LSM(opt="P",m=100,npaths = 10000, E=100,
St=100,Tmin=0,Tmax=5)
   ##American call with CVA & FVA

   vect_amc_cva=sapply(c(100:300),americanOpts_CVA_LSM, opt = "C",
   npaths=1000,m=100,E=100,Tmin=0,Tmax=5)
```

```
matplot(c(100:300),cbind(t(vect_amc_cva)[,1],t(vect_amc_cva)[,2], t(vect_
amc_cva)[,3]), type="l",pch=c(1,2,3), col = c("green", "blue", "red"), xlab
=expression(paste("Asset price ",S[t])) , ylab = expression(V[t])) legend("topleft",
legend=c(expression(paste("American Call-",V)),
expression(paste("American Call-",hat(V))),"CVA+FVA"),col=c("green", "blue",
"red"), lty=1:3, cex=0.8, box.lty=2)

# Table American call vs American call CVA & FVA

lsm_amccva=data.frame(cbind(c(100:115),t(vect_amc_cva)[c(1:16),1], t(vect_amc_
cva)[c(1:16),2],-1*t(vect_amc_cva)[c(1:16),3])) colnames(lsm_amccva)<-c("St","V",
"(V)","U")
xtable(lsm_amccva)

##American put American CVA & FVA

vect_amp_cva=sapply(c(0:200),americanOpts_CVA_LSM,opt = "P",
npaths=1000,m=100
,E=100,Tmin=0,Tmax=5)
matplot(c(0:200),cbind(t(vect_amp_cva)[,1],t(vect_amp_cva)[,2], t(vect_
amp_cva)[,3]), type="l",pch=c(1,2,3), col = c("green", "blue", "red"), xlab
=expression(paste("Asset price ",S[t])) , ylab = expression(V[t])) legend("topright",
legend=c(expression(paste("American Put-",V)),
expression(paste("American Put-",hat(V))),"CVA+FVA"),col=c("green", "blue",
"red"), lty=1:3, cex=0.8, box.lty=2)

# Table European put vs European put CVA & FVA
lsm_ampcva=data.frame(cbind(c(90:105),t(vect_amp_cva)[c(90:105),1], t(vect_
amp_cva)[c(90:105),2],-1*t(vect_amp_cva)[c(90:105),3])) colnames(lsm_ampcva)<-
c("St","V", "(V)","U")

xtable(lsm_ampcva)
```

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LEGAL CHALLENGES AND RESPONSES TO DISTRIBUTED LEDGER TECHNOLOGY (DLT) IN THE CLEARING AND SETTLEMENT OF SECURITIES. A STUDY FROM THE COLOMBIAN SECURITIES MARKET AND THE MERCADO INTEGRADO LATINO AMERICANO MILA.

Salazar Acero, Felipe

ABSTRACT

New technology has changed the way we perceive financial services today. New technology companies are taking part in a new market, offering different alternatives to allocate and channel resources through the use of apps and internet platforms. Mobile payment, issuance of securities, asset management and money transfer are some of the sectors where traditional finance services are migrating to new types of virtualized financial markets that are growing day by day. These new alternatives could bring efficiencies, as well as cost reduction in capital flows within the economy, but they also may create systemic risks thanks to the lack of regulations and supervision, which has permitted a regulatory arbitrage with high potential to affect financial market stability.

Post-trading of securities, particularly clearing and settlement infrastructure, is one of the most popular areas where the use of Distributed Ledger Technology is being contemplated, in efforts to reduce intermediations and costs in the value chain and to increase efficiencies in the entire process. This article analyses the legal implications and explores the challenges that regulators should consider when migrating to this new technology applied to the post-trading infrastructure, mainly focused on the Colombian capital market and The Mercado Integrado Latinoamericano (MILA).

Introduction

Financial markets are some of the most regulated and supervised sectors within the worldwide economy, due to their importance in delivering financing support to the different productive sectors. Humanity has witnessed devastating financial crises during the last eighty years, fostering a prevailing uncertainty and distrust throughout the whole system. Regulators and supervisors have worked together to recover investor confidence by guaranteeing the protection of their rights and savings. However, this has not been an easy task since financial innovation has gained importance and relevance in the way new capital can be channelled, from the resources providers to those who are requiring them. The creation of new complex products, and the development of new technologies used in the financial markets, are making these tasks a real challenge.

The term Financial Technology, or FinTech, has become popular in the jargon of business and its popularity has no boundaries. Basically, FinTech means the application or the use of technology-based solutions in financial services, either in the back-office or front-office. The use of these new solutions is currently changing the way in which financial services are normally presented to customers[1]; new products such as mobile payment, issuance of securities, asset management and money transfer are some examples of the expanding portfolio of financial services offered through new technologies and platforms. The vastness of new terminologies and products in this growing field makes it necessary to maintain an active and updated study of the new FinTech trends, especially on how to regulate and supervise them.

The creation of new complex products, and the development of new technologies used in the financial markets, are making these tasks a real challenge.

Since the creation of the Bitcoin by Satoshi Nakamoto in his paper "Bitcoin: A Peer-to-Peer Electronic Cash System"[2], the concept of Distributed Ledger Technology is constantly discussed as an important alternative, which may create major efficiencies in financial transactions without the need of trusted third parties. This permits the direct interaction between both sides of the deal, eliminating the intermediation and reducing transactional cost. The application of Distributed Ledger Technology to different financial services is analysed by professionals in the industry and regulators in order to update the infrastructure on which the main markets are underpinned.

[1] Bernard Marr, ´The Complete Beginner´s Guide to FinTech in 2017´ (Forbes Magazine, February 10, 2017). <https://www.forbes.com/sites/bernardma-rr/2017/02/10/a-complete-beginners-guide-to-fintech-in-2017/#79a6f2033340 > accessed 27th March 2018.

[2] Satoshi Nakamoto, " Bitcoin: A Peer-to-Peer Electronic Cash System" (www.bitcoin.org) <https://bitcoin.org/bitcoin.pdf> accessed 27th March 2018

Clearing and settlement in the capital market are areas where the application of the Distributed Ledger Technology is often considered. The Australian Stock Exchange was the first one to contemplate the adoption of this technology to record transactions and to accomplish the clearing and settlement of equities.[3] Likewise, The Society of Worldwide Interbank Financial Telecommunication (SWIFT) and a group of seven Central Securities Depositories (CSDs), have signed a Memorandum of Understanding (MoU) for a joint study of the benefits that the application of distributed ledger could bring for the post-trading of securities.[4] Although the benefits obtained from this technology seem to be clear, thanks to the high possibility of getting rid of post-trading intermediation, it remains unclear how this disintermediated infrastructure could be met in those countries where capital markets are not highly developed. Thus, intermediation will still be necessary, but with different scope in its role.

This document intends to analyse and bring forward some thoughts on the application of Distributed Ledger Technology in the Colombian Capital Market post-trading and in the "Mercado Integrado Latino Americano" (MILA). It will study the issues that can arise, and identify the challenges that market participants and regulators will have to address. The research has been divided as follows: Section One will deal with the Distributed Ledger Technology, identifying its main features and benefits for the post-trade of securities. Section Two will address the clearing and settlement of securities and its importance in the global financial market, as well as its main function in the post-trade of securities, highlighting the role of its participants. Section Three will analyse the legal and operative challenges and also risks of distributed ledger technology in the Colombian capital market. Section Four will cover the benefits, if there are any legal risks and challenges to applying the Distributed Ledger Technology in the MILA market. Finally, it concludes that the technology could bring some benefits if it is considered to be applied in the Colombian capital market infrastructure, as well as in the MILA market.

[3] David Meyer, ´The Australian Securities Exchange Just Made Blockchain History´ (Fortune Magazine December 7, 2017) <http://fortune.com/2017/12/07/blockchain-technology-australian-securities-exchange-asx/> accessed 29 March 2018.

[4] SWIFT, ´SWIFT and CSD Community Advance Blockchain for Post-Trade´ (SWIFT, Press releases January 16, 2018) <https://www.swift.com/news-events/press-releases/swift-and-csd-community-advance-blockchain-for-post-trade> accessed 29 March 2018.

01 | The Distributed Ledger Technology (Blockchain).

Definition

Distributed Ledger Technology (DLT) and Blockchain have been used indistinctly. Although both concepts may be similar in their characteristics and functionalities, they are slightly different in essence: technically, they should not be used interchangeably, since every Blockchain is a DLT, but not every DLT is a Blockchain[5]. Thus, for the purpose of this research, the concept of Blockchain is going to be analysed as a DLT, notwithstanding leaving it clear that Blockchain has its own technical features conformed by blocks added in the chain of information.

Generally, a DLT is defined as a non-centralized data base system held on a shared network in which its participants (nodes) may have their own identical copy of the ledger, allowing them to read, include, or modify information within it. Normally, this information is related to assets and properties. A Blockchain is a DLT in which each of the transactions are linked between themselves by a chain of digital blocks. Each of those blocks is attached to the others, through a cryptographic process with a digital "fingerprint" created by a functionality called “hashing”.[6] Each input of information in the chain has to include mathematically the same hash of the previous blocks in order to match them and be included in the chain of blocks.

The word Distribute makes allusion to the feature that every new record of information included, modified, or eliminated in the data base will be known and viewed immediately

[5] D Philip Treleaven, Richard Gendal Brown, and Danny Yang, ´Blockchain Technology in Finance´ (2017) 50 (9) Computer 14.
[6] Ibid

by each of the network´s nodes. Then, this allows information to be updated across the system without requiring a centralized data base validation system, permitting the process to be accelerated and improving the management of data, recording and updating it more efficiently[7]. However, this attribute raises some questions on how the information can be safely held in the network, and how the participants should be controlled at the moment of creating new inputs of information. This concern will be critical when considering whether DLT can be used on the post-trading of securities.

Security on the ledger has been recognized as the most innovative and secure system. It is underpinned by the use of cryptography[8] and keys (public and private) that guarantee to keep the information safe[9]. The use of these codes and keys depends on the consensus or protocols of validation agreed on the ledger by the participants. The consensus determines the conditions that participants should have to complain in order to validate and approve any transaction within the ledger.[10] The fact that the information is shared through the network creates many copies of the same database as nodes are participating, so any unscrupulous attacker should attack each of the copies at the same specific time.[11] The attack of one copy will be reflected in the copy of the other participants who will have to agree on the specific modification pretended on the ledger; if the node does not recognize the intended modification, he or she will reject it and the attack will become ineffective[12]. Thus, to achieve an effective cyber-attack, the attacker must affect all the copies at the same time and would have to influence the validation protocol followed by the participants.

Consensus

Since DLT has become an interesting technology to facilitate the recording and transfer of assets in the financial markets, the security standard offer by DLT is in constant analysis by regulators. Today post trading of securities is based on a centralised mechanism in which the information is controlled and validated by a central party, which is in charge of keeping, reconsolidating, and validating the unique database or gold ledger. The new features of decentralization are disrupting the traditional system used in the post-trading of securities. The consensus between the participants and the validation mechanism agreed by them will be a key aspect to ensure security in the system.

Since the information in the ledger is shared with each participant, it gives them the power to modify, include, or update information without any confirmation by trust party; it is necessary, therefore, to establish certain mechanisms for validating and confirming any modification. For example, once the DLT is set up, the participants may agree to use a consensus mechanism by which it will be possible to verify the information included in the ledger. This will determine how participants must behave in case of a cyber-attack, how

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[7] ISADA and Linklaters, ´Whitepaper Smart Contracts and Distribute Ledger - A Legal Perspective´ (August 2017) <https://www.isda.org/2017/08/03/smart-contracts-and-distributed-ledger-a-legal-perspective/> accessed 04 April 2018.

[8] Cryptography is the process of securing information that travel through internet or public networks by the use of encryption process, which turns the message into computing codes becoming it incomprehensible for the human ayes. In order to decrypt or decode the message will be required the use of a mathematical key. See Graham Goocha and Michael Williams, Dictionary of Law Enforcement (Oxford University Press 2 ed 2015).

[9] ENISA, ´Distributed Ledger Technology and Cybersecurity´ (December 2016) <https://www.enisa.europa.eu/publications/blockchain-security> accessed 05 April 2018.

[10] Deutsche Bundesbank, ´Distributed ledger technologies in payments and securities settlement: potential and risks´ (2017) vol 69 (9) Monthly Report of The Deutsche Bundesbank p- 35-49

[11] Government Office of Science, ´Distribute Ledger Technology: Beyond Blockchain´ (Research and Analysis GS/16/1 2016) <https://www.gov.uk/government/publications/distributed-ledger-technology-blackett-review> accessed 06 April 2018.

[12] Ibid.

to validate the information or how to include or eliminate data stored in the ledger.[13] The consensus will allow participants to appoint authorised nodes (Validators) able to validate any update into the ledger without the need of a centralised party.[14] In a restricted DLT the relationship between each of the nodes may be regulated under a master contract in which the scope of the validators' duties should be specified. Therefore, any modification to the ledger that doesn't observe the procedures included in it can be considered as a contract breach.[15] Once the information is validated, the participants may replicate the data and update their own ledgers in accordance with the procedures established in the consensus.

The procedure of how any updating into the ledger should be validated will be a matter of which type of consensus algorithms has been adopted. DLT permits the interaction between participants to be coordinated through two consensus algorithms known as the Probabilistic consensus algorithms (proof of work and proof of stake) and Deterministic consensus algorithms (Practical Byzantine Fault Tolerance).[16] In the first consensus option each of the validators, previously pointed out by the participants, will assess and compare the new transaction against the previous one recorded on the ledger following the rules or procedures agreed in the network.[17] Under this model, the validation may be performed with the participation of two or more validators. Each of them is allowed to include a transaction on the ledger, which should be validated by another validator, who at the end will consider if the information received is accurate and therefore can be included in the ledger. On the other hand, Deterministic consensus algorithms allows a validator leader to be appointed who will decide if the new transaction may be accepted on the ledger.[18] The benefit of this last alternative is intermediary risk reduction, since validation will not rely on the decision of two or more validators, a situation that may create inconsistencies.

The mechanism mentioned above may change the way in which the verification process and the updating of the ledger is done through the book entry managed by a centralised party, particularly in the case of the Colombian post-trading system today. The CSD is in charge of updating the book entry of the credit or debit of securities transferred according to the information delivered to it by the register systems, the trading system, the issuers, the external clearing and settlement systems and by depository participants.[19] Thus, the CSD is in charge of keeping, verifying and managing the centralised ledger that contains all the information of each of the investors' and intermediaries' accounts. It is important to mention that in virtue of the principles that govern the book entry recording in Colombia, the CSD only records transactions on its ledger by request of the owner of the securities or

[13] FCA, ´Distributed ledger technology – the FCA Discussion Paper´ (2017) DP17/3 <https://www.fca.org.uk/publication/discussion/dp17-03.pdf > accessed 06 May 2018.
[14] Ibid.
[15] Andrea Pinna and Wiebe Ruttenberg, ´Distributed ledger technologies in securities post-trading´ (2016) European Central Bank Occasional Paper Series 172, 10 – 12 < https://www.ecb.europa.eu/pub/pdf/scpops/ecbop172.en.pdf > accessed 01 January 2018.
[16] ibid.
[17] Colin Harper, ´Making Sense of Proof of Work vs Proof of Stake´ (Coincentral 24 January 2018) <https://coincentral.com/making-sense-of-proof-of-work-vs-proof-of-stake/> accessed 07 May 2018.
[18] Andrea Pinna and Wiebe Ruttenberg (n 15)
[19] Article 6 Deceval´s Operation Rule Book 2011.

the holder of the right subject of registration, or by request of authorised entity for that purpose.[20] This means that the Colombian CSD is not authorised by law to perform any registration on its own initiative. A deep discussion of this matter will be addressed in detail in Section III.

Smart Contract

Smart Contracts are part of the DLT but not every DLT has a smart contract. The legal nature of these contracts has been widely discussed, attempting to bring a common definition to establish if it can be considered equal to a legal contract. In a White Paper published in 2017 by the International Swaps and Derivatives Association (ISDA),[21] the ISDA points out that Smart Contracts are hosted in a DLT as they are used to execute certain obligations that can be codified and automatized on the ledger, such as payments, after certain conditions have been met. On the other hand, the first approach of a legal definition can be found in the Arizona House Bill 2417, signed into law on March 29 of 2017, by which the state of Arizona recognised that any signature secured in a blockchain is considered as valid electronic signature, as well as the recognition of smart contracts with legal effects, valid and enforceable.[22] In the same way, Smart Contracts are defined in the Arizona House Bill as *"an event-driven program, with state that runs on a distributed, decentralized, shared and replicated ledger and that can take custody over and instruct transfer of assets on that ledger"*. This simple description of Smart Contracts shows that those "contracts" may not be considered legal contracts as we understand today, but some jurisdictions have recognized that they could have some type of legal effects for the parties, such as enforceability. Although they are only a codification of certain obligations or clauses, this codification is just a representation of the written Master Agreement on the DLT.

This means that participants on the ledger must agree on how they are codifying the Smart Contracts in the DLT, so that the system will automatically execute the codified provisions. For this purpose, it is important to bear in mind that at the moment of drafting the legal documentation and its representation in the Smart Contract, two kinds of clauses have to be considered: i) The Operational Clauses, and ii) the Non-operational Clauses.[23] These definitions are extracted from the Arizona House Bill 2417, by which

[20] Article 2.14.1.1.1 (3) Decree 2555 of 2010 (COL).
[21] ISDA and Linklaters (n 7)
[22] Ariz. Rev. Stat. § 44-7061 (2017).
[23] ibid.

the first kind of clauses are those that contain a condition where the execution of certain obligation will depend on the occurrence of a specified event. For instance, when the provision requires an asset transferring for the payment of an amount of money previously agreed by the parties. On the other hand, the second clauses are those contractual provisions that are governing the general legal relationship between the parties. Those are the clauses that for example govern the events of controversy between the parties, the law and jurisdiction applied to the contract, the general legal principles under which parties have to behave, etc. As we can see, the Operational Clauses are those that can be expressed easily by codification in the Smart Contract, which will permit its self-execution once the condition has been met and confirmed by the nodes participating on the ledger.

The clauses mentioned above raise questions about how the Smart Contracts, DLT and the legal written contract should interact between each other to guarantee a full enforcement inside the ledger and among its participants. This harmonisation will allow the payment and transfer of assets at the same time (delivery vs payment) efficiently and securely. Hence, participants of the ledger should draft a master contract, in which all the conditions will be included as in any legal agreement; once this document is completely drafted in a common language some of the clauses will be coded and represented in a computer system and held in the ledger. This coding process will be stored on the DLT as a smart contract and it will execute the obligations once the agreed requirements have been fulfilled.

Token

As mentioned before, DLT or Blockchain permit the ownership of a certain asset in the ledger to be recorded without requiring a centralised data base. However, those assets should be represented in a way that may be recognised by the participants or nodes throughout the network and facilitate their transfer. This process has been done by coding and registering the assets in the Blockchain or DLT as digital assets or tokens.[24] This is mainly necessary for instance, when the purpose of these technologies is to raise funds by the issuance of tokens against the transfer of a certain amount of cryptocurrencies through a process called Initial Coin Offerings (ICO).[25] ICOs are a new funding alternative that allows companies interested in developing a Blockchain infrastructure to raise funds through the issuance of tokens within a blockchain platform in which they will exchange them for cryptocurrencies.[26] Although the purpose of this document is not to discuss the ICOs, the developments on regulation applied to this new alternative of funding may have an important implication on the clearing and settlement of securities around the world.

Since the ICOs were mainly used by IT companies to raise funds during 2017 and the beginning of 2018,[27] this

[24] Aron Wright and Primavera De Filippi, ´Decentralized Blockchain Technology and The Rise of Lex Criphtographia´ (2015) Cyberspace Law E- Journal, <https://www.intgovforum.org/cms/wks2015/uploads/proposal_background_paper/SS-RN-id2580664.pdf> accessed 13 June of 2018.

[25] Laura Noonan, ´Singapore Keen on Initial Coin Offerings´ Financial Times (Initial Coin Offerings November 15 2017) <https://www.ft.com/content/17173c92-c9e6-11e7-ab18-7a9fb7d6163e> accessed May 29 2018.

[26] Aurellio Guerra and Nydia Remilina, ´The Law in Finance of Initial Coin Offerings´ (2018) Ibero-American Institute for Law and Finance Working Paper <https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3182261> accessed 13 June of 2018.

[27] Ibid.

make us think that in the future companies from any sector will find it interesting to issue and record their shares through a DLT network. Thus, these digital assets have to be studied by regulators with the purpose of determining if they can be considered as securities according to each national framework. An example of this approach is the Delaware Initiative that follows the application of this new technology in three main aspects[28]: First the use of DLT in the Delaware Public Archives to improve the compliance of the maintenance and destruction of archived documents; second the use of DLT as an alternative to fill the Uniform Commercial Codes (UCC); and third the "Distributed Ledger Share" which looks to record the shares of companies in the DLT. Only the shares of a new company that the Division of Corporation has signed (cryptographically) and transferred may be consider valid throughout the DLT network. This is evidence that companies and regulators are not so far away from considering DLT technology to register new shares or even issue new securities under the concept of tokens.

For the purpose of the clearing and settlement of securities with the use of DLT it is important to define if the ledger is going to be used only for asset registration or if it is also for the issuance of new securities/tokens. The rights that underpin the tokens will determine their characteristics and whether they could be consider securities. This is important, since in some jurisdictions participants are only allowed to clear and settle assets that are considered securities. For example, in the case of Colombian securities regulation, the Law 964 of 2005 establishes that the clearing and settlement of securities is an activity of the Colombian capital markets, therefore it has to be regulated and supervised by the government.

Permissioned and Open ledger

DLT can be classified as an open ledger (non-permissioned) or closed ledger (Permissioned). This classification depends on the level of access to the network by third parties. In the same way, the security protocols will depend on the condition that the ledger will be permissioned or non-permissioned.[29] Basically, an open DLT is a system that allows any third party to access the information held on the ledger and to include, modify or update the information once the validation procedure or protocol is completed. On the other hand, the closed ledger is a network in which the access is restricted to specific and identifiable entities or participants with explicit roles.[30] Thus, any modification will only be possible if it is made by an authorised member. In a restricted ledger, the possibility

[28] Andrea Tinianow, ´Delaware Blockchain Initiative: Transforming the Foundational Infrastructure of Corporate Finance´ (2017) Harvard Law School Forum on Corporate Governance and Financial Regulation <https://corpgov.law.harvard.edu/2017/03/16/delaware-blockchain-initiative-transforming-the-foundational-infrastructure-of-corporate-finance/> accessed June 14 of 2018.
[29] Andrea Pinna and Wiebe Ruttenberg, (n 15) 10
[30] The European Central Bank, ´The Potential Impact of DLTs on Securities Post-Trading Harmonization and on The Wider EU Financial Market Integration´ (Advisory Group on Market Infrastructures for Securities and Collateral 2017) <https://www.ecb.europa.eu/paym/intro/governance/shared/pdf/201709_dlt_impact_on_harmonisation_and_integration.pdf> accessed January 20 of 2018.

of knowing who the members are permits the monitoring of the nodes´ behaviour, which guarantees that every party should be exposed, inside and outside the ledger, facilitating the accountability of the participants.[31]

Since the access to the ledger can be limited, financial institutions as well as regulators are considering the use of restricted ledger as the more appropriate one. Hence, it may be considered that in clearing and settlement processes, the use of closed ledger could be take into account to protect the network from third parties' misbehaviours and allow an effective governance among participants. By this, validation procedures will narrow the ability of an unauthorised third party to modify the information included or to be included. In the moment of including a new record on the ledger, the authorised participants that have the role of validators will determine if the new data is accurate through a consensus mechanism and the use of digital signature.[32] Once the consensus is achieved and perfectly signed by the validators, the information can be included as a new block of information.

Different approaches with regards to the use of DLT technology in the clearing and settlement of securities have been made by some authorities such as the European Central Bank,[33] the Central Bank of Japan,[34] The United Kingdom Government,[35] and the International Swaps and Derivatives Association.[36] In those cases, the authorities concluded that to preserve security, stability, and soundness of the clearing and settlement structure, the most appropriate alternative is to develop a restricted DLT. Since open and decentralised ledgers may create difficulties for governments to undertake effective control and supervision, which could become a risk for the financial system, those difficulties may potentially create new financial crises and recessions.[37] In that sense, it may be concluded that restricted ledgers will allow governments and also participants to monitor the other participants' activities through mechanisms that are underpinned by stable enforcement and governance protocols. If the restricted DLT means that only specific entities will have access to the ledger, and some of them will be appointed as validators, this make us considerer that those validators should be a trustworthy and reliable entity. Hence brokers, custodians, banks or government entities would be the best options to appoint. Now, appointing any of those entities as validators in a closed ledger seems to be contradictory to the purpose of this disruptive technology, which is the reduction or elimination of intermediaries. Investors will still have to require an intermediary that acts on their behalf towards the network. Thus, they will still not have direct and independent access to the system to clear or settle any transaction.

[31] FIbid.

[32] Government Office of Science (n 11)

[33] European Central Bank (n 30)

[34] See European Central Bank and Bank of Japan, ´ Project Stella: Securities Settlement Systems: Delivery vs Payment in a Distribute Ledger Environment´ (Joint Research Project Estella March 2018) < https://www.ecb.europa.eu/pub/pdf/other/stella_project_report_march_2018.pdf> accessed 15 April 2018.

[35] See UK Government Chief Scientific Adviser, Report of Distribute Ledger Technology: Beyond Block Chain (Government Office for Science 19 January 2016) https://www.gov.uk/government/news/distributed-ledger-technology-beyond-block-chain accessed 13 May 2017.

[36] See ISDA and Linklaters (n 7)

[37] Aron Wright and Primavera De Filippi, ´Decentralized Blockchain Technology and The Rise of Lex Criphtographia´ (2015) Cyberspace Law E- Journal, <https://www.intgovforum.org/cms/wks2015/uploads/proposal_background_paper/SSRN-id2580664.pdf > accessed 13 June of 2018

02 | Clearing and Settlement of Securities

Definition

The Payment, Clearing and Settlement Supervision Act of 2010, which amends the Title VIII of the Dodd Frank Act in the United States, defines the Clearing and Settlement activities as: The "activities undertaken by one or more financial institutions to facilitate the completion of financial transactions".[38] Each of those activities plays an important part in the post-trading of securities after the transactions have taken place. Some of the financial institution that are part of "Value Chain" are Brokers, Exchanges, Central Securities Depositories, Central Counter Parties (CCP), Custodians, and Central Banks. For example, in The United States the Clearing and Settlement are operated by Federal Reserve Banks and Privates Corporations.[39] Although this may vary in each jurisdiction because of their legal framework and infrastructure scheme, a general agreement among participants is set up to define the moment when the transaction is executed by the traders (matching) followed by the confirmation of the counterparties´ obligations, until the moment in which those obligations are fulfilled (the payment and delivery of the securities) and formalized by the recording of the transfer of ownership in the book entry accounting system managed by the CSD. All of this process, and by international standards requirements, takes three business days (T-3) to settle each transaction.[40] On the last day, the Central Securities Depositories send a credit or debit netting confirmation to the firms confirming the transfer of securities, and to the central banks, or settling banks, to proceed with the payment[41].

The Committee on Payments and Market Infrastructures (CPMI) defines the Clearing function as: "[t]he process of transmitting, reconciling, and confirming transactions previous to settlement, potentially including the netting[42] of transactions and the establishment of final positions for settlement".[43] Meanwhile, The European Parliament and The Council on the OTC Derivatives, Central Counterparties and Trade Repositories Regulation has defined clearing as the process in which the positions are established.[44] This includes the identification and calculation of the net obligations, ensuring that the securities and the cash are available to secure the settlement of the positions. On the other hand, settlement is defined by the CPMI as "[t]he discharge of an obligation in accordance with the terms of the underlying contract".[45] This occurs when the transfer of ownership is registered in the books kept by a settlement system.[46] Thus, in securities settlement the discharge of the obligations is done by the transfer of the securities to the buyer from the seller, and the transfer of the funds to the seller from the buyer. In that sense, the Uncertificated Securities Regulation 2001 in the United Kingdom points out that settlement is the delivery of securities to the transferee and the creation of the associated obligations to make the payment.

A general overview of the whole process is divided into three main stages. First the matching and confirmation; Second the clearing; and Third the settlement and custody. In each of these moments, well-organized and synchronized systems are playing an

[38] Article 7 (A) The Payment, Clearing and Settlement Supervision Act of 2010.
[39] Marc Labonte, ´Supervision of U.S. Payment, Clearing, and Settlement Systems: Designation of Financial Market Utilities (FMUs)´ (Congressional Research Service 2012) <https://fas.org/sgp/crs/misc/R41529.pdf> accessed 19 of June 2018
[40] Recommendation 3, Principles of Financial Market Infrastructure CPSS 2012.
[41] Virginia B. Morris, Stuart Z. Goldstein, Life Cycle of a Security (First Edition, Lightbulbs Press 2010).
[42] The Committee on Payment and Market Infrastructure defines netting as the offsetting of obligations owed by the participants among each other. BIS, Glosary, (2016) available at https://www.bis.org/cpmi/publ/d00b.htm?&selection=11&scope=CPMI&c=a&base=term
[43] Ibid.
[44] Regulation (EU) No 648/2012 The European Parliament and of the Council on OTC Derivatives, Central Counterparties and Trade Repositories [2012] OJ L 201/1.
[45] BIS, Glosary, (2016) available at https://www.bis.org/cpmi/publ/d00b.htm?&selection=11&scope=CPMI&c=a&base=term
[46] Dermont Turing, Clearing and Settlement (Second Edition, Bloomsbury 2016) ch Introduction to Clearing and Settlement.

important role in ensuring the appropriate performance of the capital markets. As Linciano et al. explain, the first moment begins with the trade matching.[47] At this moment the information regarding the order of buying and selling is included by the trader through an electronic order book. Once the data is included, and complemented in the interface, this information is validated and confirmed by the clearing and settlement system.

Once the information is confirmed, the clearing phase take place when the obligations owed by the counterparts are identified by the system managed by the CCPs. Only clearing members have the permission to forward the information directly to the CCP, otherwise if the intermediary does not have a clearing membership, it has to send the information through another General Clearing Member (GCM).[48] Once the information required by the clearing procedure is incorporated, the CCP novates the orders matched previously to control the counterparty risk. Hence, the CCP becomes the "buyer to every seller and the seller to every buyer" by creating a "bilateral net (cash and securities) balance" against the CCP.[49] As Turing suggests, at this stage the CCPs have an important role since those intermediaries are dealing with the counterpart risk that arises at any transaction, guaranteeing that the transaction will be settled.[50] It is important to bear in mind that once the transaction is accepted and confirmed by the clearing system, the CCP interpose itself as the buyer's and seller's counterpart by creating a multilateral netting.

After the CCP has created the multilateral netting positions, and is performing as counterpart for each member, the buyer and the seller, the discharge of the payment and the transfer of securities have to take place. This moment is refer as settlement. The Central Securities Depositories linked with the CCP receive the information of the investors, to which it has to debit and credit the securities account through the book-entry accounting system.[51] In certain cases, the CSD facilitates the transfer of funds for the payment through the payment system. For instance, in the Unites States settlement infrastructure, the National Securities Clearing Corporation (NSCC), which acts as a CCP, sends instructions to the CSD, The Depository Trust Company (DTC), to debit and credit the DTC's client securities accounts.[52] This movement on the clients' positions is made by a registering process called book-entry, as mentioned before. At the same time, the position of cash is debited and credited by the DTC, as a settlement agent trough the client´s settling bank account using the appropriate channel of settlement.[53]

As can be noticed, the definitions brought by different jurisdictions have created some uncertainties about how the clearing and settlement process should be understood. Specially, at the moment of identifying when we are in the clearing stage and when we move towards the settlement of the transaction. Many different types of clearing systems, as well as settlement of securities, are in operation in each jurisdiction. Those systems are designed to operate according to the type of securities, the kind of contracts and even the characteristics of its participants. For example, in the United Kingdom, the Clearing systems in operation are: i) The LCH´s system, which is in charge of clearing transactions over CDS, Commodities Futures and Options, Cash Equities, Repos, Currencies or Forexclear[54] ; ii) The ICE Clear

[47] N. Linciano et all, ´The Clearing and Settlement Industry, Structure, Competition and Regulatory Issue´ (2006) 58 Quaderni di Finanza 1
[48] Nasdaq, ´General Clearing Member Model´ (GCM Model Nasdaq) <https://business.nasdaq.com/trade/commodities/membership-connectivity/membership/gcm-model.html> accessed 20 June 2018.
[49] The CCP creates a bilateral net balance for each participant (seller and buyer) composed by two legs, cash and securities, in which the CCP became the creditor of each leg.
[50] Dermont Turing (n 46)
[51] Ibid.
[52] Rule 11 (g), National Securities Clearing Corporation Rules and Procedures 2018.
[53] Procedure VIII (D), ibid.
[54] BLCH Limited, ´Services´ <https://www.lch.com/services> accessed 05 June 2018.

Europe's system for clearing transactions done over interest rates, equity index, agricultural and energy derivatives, and European credit default swaps[55]; and iii) EuroCCP´s system clearing transactions with cash/equity around Europe[56]. For settlement, the transfer of funds and securities is made through different channels such as the CREST by which securities in the UK and Irish equities, government debt, corporate bonds, exchange trade funds and money market instruments are settled[57], and the Payment System of Large- Value Payment System CHAPS, Faster Payments Services (FPS) and Cheque and Credit Clearing (C&CC).[58] Since the markets are more interconnected and after the consequences brought about by the last financial crisis, it has been necessary to establish certain international standards, which ensure the counterpart risk management and the finality and irrevocability settlement of securities.[59] Although the majority of them have been adopted as recommendations and principles, the post-trading securities operators have applied them as a guide to strengthen their operations in accordance. This has created common patterns to establish a solid post-trading infrastructure, protecting the smoothness and soundness of the clearing and settlement of securities, requiring CCP and payment systems to comply with certain duties and specifications.

The Clearing and Settlement function/goals

The Clearing and Settlement infrastructure is an important architecture for financial markets that ensures the safety performance of the capital markets, permits the efficient allocation of resources as well as the development of the economy. Thanks to market interconnection that has permitted transactions to be undertaken in a cross-border scenario, the exchanges and post-trading infrastructures require the global securities market to be coordinated to be run properly, reducing costs to investors and decreasing the risks associated with it. This is plausible, if the efforts done by each jurisdiction can be aligned with the idea of developing a well-structured post trading system that permits the efficiency of transactions and the management of systemic risk.[60] However, this objective has not been easy to accomplish since legal frameworks around the world are not aligned, and has created some complexities in the performance of the modern global financial markets.

However, this objective has not been easy to accomplish since legal frameworks around the world are not aligned

[55] ICE Clears Europe, ´Clearing: ICE Clear Europe´ <https://www.theice.com/clear-europe> accessed 05 June 2018.
[56] EuroCCP, ´Services Overview´ <https://euroccp.com/home/services/overview> accessed 05 June 2018.
[57] Euroclear, ´Streamlined Real-Time Settlement´ (2012) <https://www.euroclear.com/dam/PDFs/Settlement/EUI/MA2740-CREST-settlement.pdf> accessed 05 June 2018.
[58] BIS, ´Payment, Clearing and Settlement System in The United Kingdom´ (CPSS – RedBook 2012) <https://www.bis.org/cpmi/publ/d105_uk.pdf> accessed 01 June 2018.
[59] See, BIS, ´The Core Principles of Systemically Important Payment Systems´ (January 2001) <https://www.bis.org/cpmi/publ/d43.htm> accessed 20 June 2018; BIS and IOSCO, ´Recommendations for Central Counterparties´ (November 2004), <https://www.bis.org/cpmi/publ/d64.pdf> accessed 20 June 2018; European Central Bank, Report of Standards for Securities Clearing and Settlement in The European Union´ (September 2004) <https://www.ecb.europa.eu/pub/pdf/other/escb-cesr-standardssecurities2004en.pdf?f6084065304b-16f2ccb1ec17b8a83ca7> accessed 23 June 2018.
[60] Principle 38, Objectives and Principles of Securities Regulation, OISCO (2017).

In 2001 The Governing Council of the International Institute for the Unification of Private Law (UNIDROIT) approved the work program called “Transactions on Transnational and Connected Capital Market”, by which the Council sought to unify and update the rules with regard to enhancing the stability of financial markets and cross-border complexities.[61] A number of issues were identified by the study group, among which were the rights and duties that arise between Central Securities Depositories and intermediaries.[62] The main issue was related to the intermediaries’ role as a holder of assets and the scheme of indirect holding. The Intermediary Risks were predominant in the whole process of securities holding, as well as the complexities that arose in unifying the adaptation of the book entries’ recording system, which replaced the traditional certified securities system.

One of the main concerns alludes to the applicable law in cross-border holding.[63] Thévenoz suggests the complexity in identifying the governing law to securities holding when different jurisdictions are involved in one transaction. For instance, an investor in Russia owns a Bond issued by the Russian Government and registered in the local CSD, which at the same time is used by these investors as collateral in a term loan agreement granted by a Canadian Bank. According to the author, this situation creates some doubts on how the collateral should be registered and under which jurisdiction. Should it be done in Russia? This, since it is the place where the bonds were registered and also the place where the relevant account of the debtor is located. Or should it be done in the Canadian CSD’s book-entries, since the creditor is a Canadian entity? This problem could be more obvious in the event in which one of those jurisdictions does not merge to the book entry scheme, and it still using the certificated securities system. In this regard, the Geneva Securities Convention sought to harmonise the rules to create a compatible international framework that governs the structure of the holding and transfer of securities; to facilitate the cross-border securities transactions reducing systemic risk and promoting market efficiency.

On the other hand, Schwarcz identifies another issue to be solved in the harmonisation of post-trading law linked with the indirect holding system of securities.[64] According to the Professor, the issue is presented in the context of cross-border transaction in which the intermediaries are international organisations. Those intermediaries are in charge of an indirect holding system by which they hold securities on behalf of their customers, but also own some rights in those assets. In this context, if an intermediary is requested by its creditors, the investors that hold an interest in the intermediary´s securities could see their assets (participation) affected by the claims of those creditors. This is not a minor concern, since investors can celebrate any other transaction over those securities with different investors or intermediaries that may jeopardise the market by creating a systemic risk. Thus, the harmonisation of the holding securities system had to be addressed to avoid bringing instability to the whole market.

[61] Unidroit, ´Transactions on Transnational and Connected Capital Market´ (Harmonised Substantive Rules for the Use of Securities Held With Intermediaries as Collateral June 2002) <https://www.unidroit.org/english/documents/2002/study78/s-78-001-e.pdf> accessed 07 June 2018

[62] Unidroit, ´Restricted Study Group on Item 1 of the Project: Harmonised Substantive Rules for the Use of Securities Held with Intermediaries as Collateral´ (13 September 2002) https://www.unidroit.org/english/documents/2002/study78/s-78-005-e.pdf accessed 07 June 2018

[63] Luc Thévenoz, ´The Geneva Securities Convention: Objectives, History, and Guiding Principles´ in Pierre-Henri Conac, Urisch Segna, Luc Thévenoz (eds) The Impact of the Geneva Securities Convention and the Future European Legislation (New York, Cambridge University Press 2013) 3-21.

[64] Steven L. Schwarcz, ´Intermediary Risk in the Indirect Holding System for Securities´ (2002) 12 (2) Duke Journal of Comparative & International Law 309.

In the same vein, an appropriate national framework must be aligned, facilitating the development of today's globalised market. In this regard, the Title VIII of the Dodd Frank Act that addresses the federal regulatory landscape of the payment, clearing, and settlement system in the United States recognised the clearing and settlement systemic importance. The Payment, Clearing, and Settlement Supervision Act of 2010 includes the concept of Financial Market Utility (FMU), that is extended to "any person that manages or operates a multilateral system for the purpose of transferring, clearing, or settling payment, securities, or other financial transaction among financial institutions". Hence, in 2012 the Financial Stability Oversight Council entitled eight FMUs as systemically important in which were included The Clearing House Payments Company, The Depository Trust Company (DTC) and The National Securities Clearing Corporations (NSCC).[65] This is a recognition that the definition of FMU underpins the idea that those entities that perform multilateral payments, clearing or settlement activities may create a new range of risks, and thus regulators may regulate and supervise them to guarantee the financial market's safety and soundness.

Also, in the United Kingdom the Financial Market Infrastructures (FMIs) are recognised as critically important in the financial markets, since they provide services that may affect the whole financial market.[66] The FMI's supervision is the responsibility of the Bank of England under five main pillars (Governance, managing operational risk, ensuring continuity of service and managing participant default) that are looking to control a potential failure of those entities that could be hazardous for financial stability.[67] For this reason, the Bank of England is in charge to guarantee that the FMIs' infrastructure operates and is managed in line with the public interest by the imposition of rules and policies designed to reduce systemic risk.

[65] UBIS, ´Payment, Clearing and Settlement System in the United States´ (CPSS-Red Book 2012) <https://www.bis.org/cpmi/publ/d105_us.pdf> accessed 11 June 2018.

[66] The Financial Market Infrastructure make reference to securities settlement systems, central counterparties and recognised payment systems.

[67] Bank of England, ´The Bank of England's Approach to the Supervision of Financial Market Infrastructures´ (April 2013) <https://www.bankofengland.co.uk/-/media/boe/files/financial-stability/financial-market-infrastructure-supervision/the-boe-approach-to-the-supervision-of-fmi.pdf?la=en&hash=CD95F6E8C-2093172F4EA183E0A552D815FAAB5C5> accessed 6 June 2018.

For the reasons expressed before and making references to how different jurisdictions have approached the supervision and regulation of the payment, clearing and settlement of securities, it is clear therefore to suggest that this infrastructure is an activity that cannot pass unnoticed. The high interconnectedness of the markets that relies on the performance of each of its participants implies that the stability of capital markets could be threatened by the systemic risk. In order to solve this, the Financial Stability Board (FSB) in 2010 issued a number of recommendations to enhance the financial market infrastructure, seeking to control the factors that could have the ability to trigger a systemic risk.[68] Under these recommendations the FSB defines Global Systemically Important Financial Institutions (G-SIFI) as those entities globally interconnected in which their distress or failure can have significant consequences in the global financial system with adverse consequences across the world. However, following Lastra's suggestion, the concept of Systemically Important Financial Institutions (SIFI) will change as the concept of systemic risk varies.[69] Bearing this in mind, new technology development has helped an interconnection between different markets, but on the other hand, it has created an extended network which relies on the behaviour of each of its participants. These new technological developments will make regulators redefine the SIFIs' catalogue by including new entities as systemically important.

The use of DLT in the market infrastructure is an example of how new technology could offer a viable alternative to create efficiencies in the clearing and settlement of securities processes, but also to redefine the categorisation of SIFI. This distributed network allows each participant to rely on each other without requiring a centralised trust party. Thus, this technology is considered as a disruptive mechanism with the potential to change the manner in which the current post-trading of securities operates, offering the reduction, or even elimination of any intermediation. Although financial intermediaries are an important part within the clearing and settlement of securities architecture, since they allow markets to perform smoothly and soundly by providing information, facilitating the transfer of funds and assets for settlement of obligations, keeping the record of ownership, doing the custody of assets, and so forth. These agents can also fail, affecting capital markets' stability and liquidity with tremendous consequences for the whole economy.

The use of DLT within financial services, mainly in the post-trading flow, will bring some challenges which regulators must address by identifying new risks with the potential to distress market stability, and also identifying new regulation which looks to protect investors. Also, the interesting point here is to analyse if in a real scenario a peer to peer technology can also eliminate the counterpart risk, the operational risk, the legal risk and might also guarantee efficiencies in the delivery and payment of securities and cash. I consider that these new companies that provide the infrastructure for DLT/Blockchain have the potential to become systemically important in the capital market, and regulators will have to be cautious on how they supervise and regulate the activity of these new market participants.

[68] UFSB, Report of Reducing the Moral Hazard Posed by Systemically Important Financial Institutions (FSB Recommendations and Time Lines, 20 October 2010) <http://www.fsb.org/wp-content/uploads/r_101111a.pdf?page_moved=1> accessed 27 June 2018.

[69] Rosa María Lastra, ´Systemic Risk, SIFI and Financial Stability´ (2011) 6 (2) Capital Markets Law Journal 197-213.

The Central Securities Depository

The analysis of the application of DLT in the post-trading of securities has opened the possibility of considering that CSDs could disappear, since their services will become obsolete. The principal argument on which the above affirmation is based, suggests that DLT functionality can eliminate the need for a centralised party, due to the fact that in a DLT environment investors, issuers, and traders will interact directly without requiring any trust party that validates and undertakes the reconciliation of any transaction, activity that is performed today by Central Securities Depositories. Any transaction will be validated and confirmed by the users or nodes, according to the consensus established by the network's protocol. Also, it would be possible to achieve the settlement in real-time, bringing efficiencies in the whole system's value chain, thanks to the reduction of time taken to settle a transaction that today is standardised in T+3. On the other hand, Philipp Peach[70] suggests, that thanks to the elimination of intermediation in DLT environment, this could bring more liquidity within the market since retail investors will be able to interact directly among themselves. With this possibility, investors will benefit from the reduction of transactional cost without requiring an intermediary who does the bridge between sellers and buyers.

However, this consideration has not been completely accepted by regulators and markets participants, since there are still some risks that have to be addressed before considering the complete elimination of intermediaries and the adoption of this technology. Indeed there is a consensus in the idea that DLT may enhance the efficiency and transparency of the post-trading infrastructure, which will change the role of the CSD. As we will develop in the following lines, the structures and procedures that today Depositories are following to execute their functions may move forward to better, and more efficient practises. Some of those changes could be narrowed to the way of how today CSDs are keeping the record of ownership of securities; the custody of dematerialised assets to a new concept of tokenized securities; the use of cryptography and private keys; the modification of the well-known book entry system to merger in the idea of a decentralised data base and the use of wallets as a new way of investors accounts.

CSD is a financial market infrastructure (FMI) in charge of the settlement and safe-keeping of financial instruments; such as equities, bonds, money markets, or any other instrument authorised by law.[71] It is important to mention that to facilitate those tasks, CSDs may immobilise or dematerialise the paper-based securities by representing them in a centralized digital record, allowing the transferring of securities under a book-entry system.[72] Also, this electronic record has helped to reduce the risk associated with the physical certificates regarding loss/theft, physical damage, and delays in the delivery of the certificates. Moreover, the book-entry accounting system has reduced the cost associated with the custody of thousands of certificates and has increased the efficiency in the settlement of the transaction, making it faster and more secure.[73] Thus, the features that make CSDs so special are because of their specific roles in the post-trading. For instance, the European Parliament has defined a CSD as an entity in charge of the operation of the securities

[70] Philipp Peach, "Integration Global Blockchain Securities Settlement With Law Regulation – Policy Considerations and International Principles" (2016 Social Science Research Network)< http://ssrn.com/abstract=2792639> accessed 15 October 2018.

[71] Keith Dickinson, Financial Market Operations Management, (Chichester, West Sussex 1st edn, United Kingdom, John Wiley and Sons, Inc, 2015) ch 6.

[72] Alistair Milne, ´Central Securities Depositories and Securities Clearing and Settlement: Business Practice and Public Policy Concerns´ in Diehl, M et al (eds), Analyzing the Economics of Financial Market Infrastructures (IGI Global 2016), 334

[73] Keith Dickinson (n 71)

settlement system, performing the record of securities ownership in a book-entry system, and providing securities accounts.[74] Thanks to technology improvements and to the electronic recording of ownership, nowadays it is possible to purchase and sell securities faster and more securely in local and cross-border markets, helping both issuers and investors looking for portfolio diversification.

All securities markets in the world must have an infrastructure that allows efficient and secure securities transactions, including all or some of the stages discussed before, ensuring investor protection and market stability. It is clear that the architecture can vary in each jurisdiction, where different operative standards could create conflicts in cross-border transactions, requiring therefore common rules. For this reason, in 1989 the Group of Thirty (G30) issued the first report in this regard with the purpose of harmonising the practices and standards in the clearing and settlement activities among the principal markets in the world, looking to simplify the process by reducing costs, inefficiencies and eliminating risks.[75] At that moment, the G30 identified a number of inefficiencies in the global clearing and settlement infrastructure, such as: I) lack of compatibility in the confirmation and trading on a local and international basis; II) the settlement of transaction was performed in different periods; III) absence of delivery versus payment DVP; IV) contradictions in the trade collaterals; and V) book-entry was not used by every settlement structure. Within the recommendations mentioned in the report, it is important to highlight the relevance given to the CSD in relation to its function in immobilising or dematerialising the securities by a book-entry registration. Later in 2003, a second report was issued by G30 in which they pointed out twenty recommendations to establish common technological and operational standards to permit, among other concerns, the operational and legal uniform standards that would allow an efficient cross-border clearing and settlement network with direct interaction among payment systems and foreign-exchanges.[76] Finally, in 2006 the Final Monitoring Report was issued by the G30, in which some aspects were mentioned that still need to be enhanced.[77] Some of them were the immobilisation and dematerialisation of securities which had some legal restrictions that required a reform in the national legal frameworks, and also difficulties establishing communication standards compatible in other jurisdictions.[78] As we can see, the main concern for securities cross-border standardisation has been the difficulty of rules harmonization, and setting up common technological communication protocols to execute the settlement of cross-border transaction efficiently and securely.

[74] Article 2 (1) of Regulation (EU) No 909/2014 of 23 July 2014 on Improving securities settlement in the European Union and on central securities depositories and amending Directives 98/26/EC and 2014/65/EU and Regulation (EU) No 236/2012 OJ 257/1 (Regulation on improving securities settlement in the European Union and on central securities depositories)
[75] Group of Thirty, Report of Clearing and Settlement System in the World's Securities Markets (Securities Clearance and Settlement Study 1989) <http://group30.org/publications/detail/49> accessed 09 July 2018.
[76] Group of Thirty, Report of Clearing and Settlement a Plan of Action (2003) <http://group30.org/publications/detail/123> accessed 09 July 2018.
[77] Group of Thirty, Report of Global Clearing and Settlement Final Monitoring Report (2006) <http://group30.org/publications/detail/134> accessed 09 July 2018.
[78] Ibid.

DLT seems to be the answer to accomplish the simplification of the post-trading of securities. Different studies have been done to determine the implications of DLT in this process. For instance, The Consortium Group of Central Securities Depositories conformed by the biggest CSDs was setup in 2017, to undertake shared research to identify how DLT could be used in the post-trading of securities, and how the current standards could coexist with the new technology.[79] In an official report released in October 2018, the working group suggested that functions of Financial Infrastructure providers, such as CSDs, will remain important as part of the whole process, but with some changes[80] . For example, Central Securities Depositories in a distributed ledger will guarantee that settlement of transaction may be legally final through the establishment of contractual agreement executed by smart contracts. Additionally, CSD's notary function will change to guarantee the matching between the number of securities tokens issued and the number of securities tokens held in the investors' accounts (digital wallets).
This is possible if the CSDs interact in a private permissioned ledger as the manager or "governor" with the power to supervise the behaviour of the network's participants.

[79] Michael del Castillo, ´The World's Largest CSDs Are Forming a New Blockchain Consortium´ (June 5 of 2017)
Coindesk <https://www.coindesk.com/worlds-largest-csds-forming-new-blockchain-consortium/> accessed 25 December 2017.
[80] International Service Securities Association, 'Infrastructure for Crypto-Assets: A Review by Infrastructure

It is possible to understand that the industry does not consider that CSD may disappear, but it will change the way it operates. In a report issued by Euroclear, the financial service company suggested that CSDs will not vanish from the post-trading of securities because of the adoption of the DLT. [81] Arguing that the application of this technology could bring more efficiencies in the process, and that CSDs will be able to control the network, because DLT could be designed in a way that they (CSDs) may participate as manager members, which will permit them to oversight the ledger. Additionally, the report concludes that CSDs could contribute in the DLT settlement infrastructure by acting as managers or custodians of the private keys and smart contracts, rather than the safe keeping of securities. This by taking responsibility in monitoring the network's operation.

On the other hand, The European Central Securities Depositories Association (ECSDA) considers that DLT may bring some improvements to the markets, through the reduction of transactional costs immersed in the securities' life cycle, as well as enhancing the transparency during the transactions.[82] Thus, the association considered that the first approach to DLT by CSDs should be done in the processing and storage of data. By storing data in the DLT, it could simplify the way to identify investors, since the identification process would not be done from one intermediary to another. However, it will be necessary to develop protocols to allow the interoperability between the current system and DLT. Although DLT could improve the functionality of the CSDs, considering its abolition is unacceptable.

[81] Euroclear, and Slaughter and May, ´Blockchain settlement Regulation, innovation and application´ (November 2016) <https://www.euroclear.com/dam/PDFs/Blockchain/MA3880%20Blockchain%20S&M%209NOV2016.pdf> accessed 03 July 2018.
[82] ECSDA, ´European Central Securities Depository Association Response to the European Commission Consultation on FinTech´ (15 June 2017) <https://ecsda.eu/wp-content/uploads/2017_06_15_ECSDA_FinTech.pdf> accessed 30 June 2018.

In this regard, Marc Robert from Clearstream pointed out that the current market infrastructure is designed in a certain way so that it brings high levels of safeness and legal certainty, which a decentralised network is unlikely to accomplish. Therefore, the elimination of intermediaries will threaten the stability that they offer to the market.[83] For this reason, and following Meijer's point of view in his article "What future role for CSDs in blockchain post-trade environment?", CSDs will still exist by adapting their core system to the new technology, which will change in some way that they are conceived today and it will create new opportunities for them to increase their portfolio of services.[84] Therefore, it is appropriate to consider DLT as an alternative to improve capital markets settlement infrastructure as it will become more efficient and faster. Moreover, this technology will open the door to increase cross-border transactions, and, at the same time, enhance the incentives for issuers to undertake more IPOs, rising the number of competitors and developing local markets.

Principle of Finality

To analyse any type of settlement infrastructure, it is important to address the finality principle. In the case of Capital Markets, this principle is fundamental to ensure certainty in the completion of transactions undertaken in the market. This is specially the case when these transactions are settled in interconnected systems located in different jurisdictions, which has increased substantially cross-border trading volumes, with the settlement system becoming a source of systemic risk.[85] For this reason, systems must interact between each other in a coordinated matter, guaranteeing confidence in the market. IOSCO and the Committee on Payment and Settlement System (CPSS) released in 2001 a joint document where the authorities issued several recommendations regarding the securities settlement systems. Addressing the legal risk by recommending rules, procedures, regulation and contractual provision that govern the securities settlement system was one of the main concerns in this document, as it must define clearly the moment and timing in which the finality of settlement should take place[86]. To analyse the implication of DLT in the settlement finality of securities and for the purpose of this document, it is therefore necessary to define the finality principle.

Finality principle has been defined by the Committee on Payments and Market Infrastructure (CPMI) as: "the legally defined moment at which the transfer of an asset or financial instrument is irrevocable and unconditional and not susceptible to being unwound following the bankruptcy or insolvency of a participant".[87] As we may notice from the definition, the principle of finality is a measure to protect the transfer of securities within a settlement system. It shows that there is not any possibility that the transaction could be affected once the finality of the transaction has been achieved. The important issue is to clearly define the exact point in time where the settlement finality is crystallised in a specific settlement system. In the EU, the Central Securities Depository Regulation (CSDR) suggests in article 39 that the settlement systems must define the stages in which transactions have entered into it, as well as when they are understood to be irrevocable. Additionally, the Settlement Finality Directive establishes that the order of transferring securities must be enforceable, according to the rules of each system[88] and, therefore, it cannot be revoked.[89] Likewise, in Colombia, the finality principle is regulated by the Law 964 of 2005 by which a transfer order is

[83] Marc Robert-Nicoud, ´Is Blockchain Technology Really the Solution To All Our Problems?´ <http://www.clearstream.com/blob/79108/fb21cbfa90082092fc-6635f1f6ffbac0/fintech-mrn-1512-data.pdf> accessed 30 June 2018.
[84] Carlo R.W. De Meijer, ´What future role for CSDs in blockchain post-trade environment?´ (Experfy May 15 of 2018) <https://www.experfy.com/blog/what-future-role-for-csds-in-blockchain-post-trade-environment> accessed 03 July 2018.
[85] Committee on Payment and Settlement System, and OISCO, Report of The Recommendation for Securities Settlement System (CPSS-IOSCO Joint Task Force on Securities Settlement System January 2001) <https://www.bis.org/cpmi/publ/d42.pdf> accessed 28 June 2018.
[86] Recommendation 1 and 8, Ibid.
[87] CPMI, ´Distributed Ledger Technology in Payment, Clearing and Settlement´ (BIS February 2017) <https://www.bis.org/cpmi/publ/d157.pdf> accessed 25 June 2018.
[88] Article 3, Regulation on improving securities settlement in the European Union and on central securities depositories
[89] Article 5, ibid

irrevocable and enforceable once it has been accepted by the settlement system, in accordance with its procedures established in the rule book[90]. Each of the existing settlement systems could create legal and systemic risk if they do not operate in a coordinated basis, since they could create uncertainties in the enforceability of the transfer orders, especially in cross-border transactions where different legal traditions could be in conflict.

Following CPMI´s recommendations and the purpose pursued by Unidroit to promote the certainty in capital markets with the establishment of technological and operational standards for the clearing and settlement process, it is appropriate to analyse those implications that DLT may have in the settlement finality. Considering that this technology is an alternative to allow the recording of ownership, as well as to automatize contracts in a decentralized network, bringing efficiencies and cost reductions in the whole process. Yet there are several challenges and risks that should be considered. In a report published in 2017 by the CPMI, the committee concluded that it is not properly clear the stage in the network where the finality could take place in a DLT environment, a situation that might bring some conflicts with domestic laws when considering the order transfer as irrevocable.[91] Similarly, the Bank of England has considered that this situation creates more ambiguity in a DLT infrastructure, when the only possibility of determining that the transaction has ended is by "probabilistic finality". [92] This type of finality refers to the forking of the ledger, by which the chain of data has been bifurcated in two or more new chains. This is solved by the protocol agreed among the participants, within which they will establish that the longest chain remains as the valid one. Although this uncertainty creates risks in the application of DLT, Euroclear supports the idea that in a permissioned DLT, the finality can be clearly defined when designing the protocol and architecture of the network.[93] At this stage participants may agree the roles that each of the participants will have, as well as the moment in which the transactions have to be considered irrevocable.

This analysis makes us consider DLT as a great tool with important benefits for the post-trading of securities, reducing operational costs and creating efficiencies in the value chain. Although the technology is still new and there is still much to do, it is possible to believe that a permissioned or restricted ledger could bring a number of opportunities to improve the whole market. The possibility of restricting the access to certain entities and to define their roles and duties is an important aspect to consider before this technology could be acceptable for regulators and governments. Regulators and supervisors cannot lose, in any circumstance, the continuous control and supervision within the network.

[90] Article 10 Law 964 of 2005.
[91] CPMI (n 85)
[92] Evangelos Benos, Rodney Garratt, and Pedro Gurrola-Perez, ´The Economics of Distributed Ledger Technology for Securities Settlement´ (Bank of England Staff Working Paper No 670 August 2017) <https://www.bankofengland.co.uk/-/media/boe/files/working-paper/2017/the-economics-of-distributed-ledger-technology-for-securities-settlement.f?la=en&hash=17895E1C1FEC86D37E12E4BE63BA9D9741577FE5> accessed 01 July 2017.
[93] Euroclear, and Slaughter and May (n 79)

03 | Distributed Ledger in Colombia

It has been described, in general terms, how the clearing and settlement operates in the securities' post-trading. Also, we have mentioned the most important entities or FMI that interact in the whole value chain. This chapter will discuss how the Colombian clearing and settlement legal framework should be adjusted if DLT is used in the securities market. The objective in this section is to mention the main regulatory challenges and adjustments that the Colombian regulator should consider. Although financial innovation in Colombia is at an early stage and there is still much regulatory analysis that has to be done, the author hopes that this document will contribute to this task.

Regarding the discussion proposed, we need to focus on the main legal framework that governs clearing and settlement of securities in Colombia. Those regulations are: The Law 964 of 2005, The Decree 2555 of 2010, and The Rule Books of each entity (FMI) that governs each stage of the process. Although this enumeration is not exhaustive, some other regulations and guidance may be considered, since they can be used as a guide for a more comprehensive approach to the purpose of this document.

The Law 964 of 2005 (Law 964) is the general statute that governs the Colombian securities market, promoting economic development, market efficiency, and investor protection, within a regulation and supervision architecture that promotes technology innovation.[94] In this regard, the

[94] Exposición de motivos al Proyecto de ley del Mercado de Publico de Valores, Gaceta del Congreso No 261, 16 de mayo de 2005.

Government regulates and supervises activities that involves the use, investment, and exploitation of investors' resources through securities (valores) as a vehicle.[95] The reason for this special treatment is brought from the Colombian Constitution, in which the stock market is considered as public interest and, as such, any person or entity interested in undertaking any activity related to it, requires authorisation issued by the state.[96] Although stock markets are mainly performed as a private activity, governed under private and commercial law, their special regulation is justified as a means of protecting investors and financial stability. Financial institutions have a prevalence position against customers, thanks to the asymmetry of information that these institutions pose, and they could use this dominant position to their own benefit by inappropriate use of investor funds affecting their interest, and the stability of financial markets.[97] For this reason, an appropriate framework has to be set up to ensure "adequate licensing procedures, strong supervisory techniques, stable liquidity and capital requirements, and accountability".[98] Thus, the Colombian regulator has established a prudent legal framework, and supervisory procedures to guarantee the stability, soundness, and confidence of the markets.

Financial institutions have a prevalence position against customers

[95] Article 1 Law 964 of 2005.
[96] Article 335 Colombian Constitution 1991.
[97] Luis Fernando López Roca, ´El principio de Igualdad en la Actividad Financiera´ (First Edition, Universidad Externado de Colombia 2012).
[98] Rosa María Lastra, International Financial and Monetary Law (Second Edition, Oxford University Press 2015) ch 3 111 - 146

Capital Markets are the scenario in which a number of institutions, trading mechanisms and procedures interact to permit securities investment.[99] The clearing and settlement is one of the capital Market's activities that facilitates capital allocation. This activity can be done by private companies incorporated in Colombia.[100] So, domestic regulation opens the opportunity to private entities to provide clearing and settlement services.[101] Although regulation allows any private entity to offer these services, they are required to fulfil a number of legal requirements before being authorised to operate.[102] In this sense, if any company is interested in offering clearing and settlement of securities services through DLT, it will have to fulfil the requirements that are established for that purpose, such as minimum capital requirements, type of corporation structure, registration in the Market Agents Register (Registro de Agentes del Mercado de Valores is its name in Spanish), and be granted the authorisation issued by the Colombian Financial Superintendence.[103] As we can see, in the Colombian framework there is no prohibition that limits any private company developing and operating a DLT infrastructure within the market. Colombia has a very friendly regulation for those who are interested in developing new alternatives that simplify financial services. However, from this writer's point of view, the current conditions on which authorisation relies could be to a certain extent expensive and with long bureaucratic procedures that may reduce the interest of those entities attracted to the idea of offering financial innovation.

We can argue that thanks to financial innovation, competition in the Colombian capital markets post-trading may increase. The possibility of a new technology offering services in the clearing and settlement of securities, allows efficiencies and cost reductions in the whole market.[104] Hence it is necessary that prudential regulation and financial innovation work together in order to facilitate market access to new players. Indeed, we should not forget that capital Market's activities are really sensitive and financial stability relies on how the markets operate which, at the same time, will define the investor's level of confidence. Therefore, those that are considering offering services to the Colombian securities market through DLT or Blockchain solutions, will have to bear in mind that their activity will be supervised, and also they must comply with regulatory requirements. This does not mean that regulators should increase the requirements, creating barriers and limiting the access of new players.

[99] Colombian Financial Superintendence, ´Basic Capital Market´s Concepts´ (December 2008) <https://www.superfinanciera.gov.co/SFCant/ConsumidorFinanciero/conceptosbasicosmv.pdf> accessed 16 July 2018.
[100] Article 3 Law 964 of 2005.
[101] German Abella Abondano, ´Sistemas de Negociación y de Registro de Valores y de Compensación y Liquidación de Valores´ (2008) Revista de Derecho Privado Universidad de los Andes.
[102] Article 2.12.1.1.1 Decree 2555 of 2010.
[103] Article 53 Decree 663 of 1993. Estatuto del Sistema Financiero.
[104] Evangelos Benos, Rodney Garratt, and Pedro Gurrola-Perez (n 90)

The concept of Clearing and Settlement of securities in Colombia is really broad. The definition suggests that this system underpins those activities, agreements, participants, rules, procedures and mechanisms that are set up to confirm, clear, and settle securities´ transactions.[105] As explained before in Section II, clearing of securities in Colombia, as well as in other market infrastructures, is understood as the moment in which the counterparties' obligations are identified (delivery of securities and transfer of cash); thus, settlement is the stage at which the counterparties' obligations are fulfilled.[106] For this purpose, entities such as the Colombian Stock Market (BVC); The Colombian Central Securities Depository (DECEVAL), and the Colombian Central Counter Party (CRCC), interact among each other through a number of procedures that are established to clear and settle transactions. For instance, the BVC manage three main trading systems for each type of assets[107] : equity, fix incomes, and Derivatives; that, at the same time, are linked with each Broker, DECEVAL and CRCC´s core system[108] to facilitate transactions' fulfilment. Each of those interconnected channels, either the trading or the post-trading, has their own regulated procedure that governs how members should interact and behave within each structure. This regulated environment guarantees that each transaction is effectively finalized, reducing the counterparty risk, liquidity risk, legal risk, and systemic risk.

An interesting aspect of those regulated systems is that they act as an exclusive club whose access is reserved to certain authorised entities. The participants have to satisfy some previous requirements or conditions that are incorporated in each entity's rule book and designed to reduce every possible risk that could rise threatening market stability.[109] This is one of the aspects where the Colombian regulator has to pay major attention in DLT adoption. Indeed, today's Colombian market, as well as international markets, is designed in a complex architecture with a broad number of intermediaries relying on a very small number of financial infrastructure entities. This situation may increase aspects such as counterparty risk and transactional cost for investors, by the creation of oligopolistic competition[110], essentially when the market is concentrated in a reduced number of market players able to offer post-trading services.[111] DLT brings new proposals offering the possibility to eliminate intermediation, where both issuers and investors may interact directly without affecting market liquidity and legal certainty, thanks to protocols and a consensus mechanism designed to validate transactions. Moreover, with the possibility of self-enforcement offered by smart contracts through their coding process by which obligations are executed directly.[112]

Although disintermediation sounds attractive to the market since it could reduce cost and create efficiencies, I consider that in order to protect investors and preserve market stability, the intermediaries will not disappear from the Colombian market infrastructure. The intermediaries play an important role in the market, they facilitate the flow of relevant information between markets and investors, as well as performing adequately the post-trading course, since they are highly trained market professionals who facilitate capital allocation.[113] Additionally, they have the appropriate information and technological infrastructure that guarantee the smooth running and stability of the markets. Otherwise, it will not be possible to undertake investments in international markets and develop financial products that are at the same level with those traded in big marketplaces nowadays. I consider that the problem with intermediation is strongly related to how the intermediate behaves (an aspect that has to be tackled by miss-conduct regulation) and, therefore, with appropriate corporate governance principles. Financial innovation is a tool that has to be embraced, especially because it allows how post-trading operates today to be enhanced. Hence the Colombian market could think of using DLT technology as an alternative to be more competitive internationally, to attract more investors as well as new issuers, which may contribute to economic development. As explained in Section I, DLT can be designed as a permissioned or restricted ledger in which each participant's role has to be clearly defined in each protocol. Thus, a good alternative may be that CSD could become the manager of the ledger, being responsible for the network´s oversight and only giving ledger access to its direct participants. This is possible today, since the Law 964 of 2005 and the Decree 2555 of 2010 do not have any limitation that restricts any new potential technological environment designed to be used in the clearing and settlement of securities, and also because it allows each entity to be self-regulated by their own rule-books.

[105] Article 9 Law 964 of 2005.
[106] Carlos Fradique-Méndez, ´Guía del Mercado de Valores´ (Bolsa de Valores de Colombia 2014) <https://www.bvc.com.co/pps/tibco/portalbvc/Home/Empresas/Guia_Mercado_Valores?com.tibco.ps.pagesvc.action=updateRenderState&rp.currentDocumentID=-7ca0c036_147b6b20b27_5e970a0a600b&rp.revisionNumber=1&rp.attachmentPropertyName=Attachment&com.tibco.ps.pagesvc.targetPage=1f9a1c33_132040fa022_-78750a0a600b&com.tibco.ps.pagesvc.mode=resource&rp.redirectPage=-1f9a1c33_132040fa022_-787e0a0a600b> accessed 18 July 2018.
[107] ibid.
[108] Article 3.5.5.1, Bolsa de Valores de Colombia´s General Rule Book. (Reglamento General de la Bolsa de Valores de Colombia 2018) accessed 18 July 2018.
[109] Article 2.12.1.1.2 Decree 2555 of 2010.
[110] John Armor, et al. ´Payment and Settlement Systems" in John Armor, et al (ed), Principles of Financial Regulation (Oxford University Press 2016).
[111] Dinero, ´Bolsa de Valores de Colombia Adquirió el 82,2% de las Acciones de Deceval´ (2017) <https://www.dinero.com/inversionistas/articulo/integracion-de-la-bolsa-de-valores-de-colombia-y-deceval/253494> accessed 18 July 2018.
[112] Philipp Paech, ´Securities Intermediation and the Blockchain: An Inevitable Choice Between Liquidity and Legal Certainty´ (2016) Uniform Law Review 21 (4) 612.
[113] Alessio M. Pacces, ´Financial Intermediation in the Securities Markets Law and Economics of Conduct of Business Regulation´ (2000) 20 International Review of Law and Economics 479

Another concern that rises with the idea in adopting DLT in post-trading of securities is how this new technology could be harmonised with securities safe-keeping and ownership recording. In Colombia, the legal framework is based on a civil law tradition by which nominative securities ownership is required by law to be recorded in the issuer's book[114]. However, since markets are highly interconnected and technology has allowed the use of paper to disappear, book recording has turned into dematerialised securities, electronically represented through a book entry accounting system.[115] As a result of this technological improvement, in Colombia securities ownership is done and recognized today as legally enforceable through book entry inputs made by a CSD in the respective investor's account.[116] It should be noted that the recognition of legal and enforceable property is conditioned by a recording process managed by authorised entity, in this case a central security depository. For this reason, today the majority of securities are issued in dematerialized basis, and the property rights are only enforceable if they are recorded by this managed electronic recording system managed.

At the moment, there is no harmonisation between the current legal ownership and safe-keeping regulation with the manner in which DLT could perform them. It has been explained before that DLT is an alternative to record information in a shared decentralized ledger, mainly used to record assets (tokens), transactions and ownership. Under this infrastructure, each of the network's participants have their own copy of the ledger, without requiring an intermediary intervention to have access to their investment account information. This information is available directly in each participant's server. In Colombia, under the current system, securities are held electronically, or physically by the CSD, who credits or debits the account designated to a direct participant, in which the securities are recognised.[117] Thus, investors can only have access to their investment account information through an intermediary, who on behalf of each of its clients has direct access to the information held in the CSD[118]. As we can see, the application of this technology will allow investors to interact directly at the moment of undertaking a transaction within the DLT reducing transactional cost and bringing efficiency to the process.

Considering that in Colombia it would be plausible to migrate the current system to DLT, it is appropriate to mention some of the alternatives that may be considered: First, as mentioned before, DLT can be designed as an open ledger or closed ledger. It will depend on how limited access to the network is. Since clearing and settlement of securities is one of the core aspects of capital markets, and thus has to be highly regulated and supervised to guarantee the market's

[114] Article 648 Colombian Commercial Code
[115] Unidroit, ´Guide on Intermediated Securities´ (16 November of 2017) International Institute for Unification of Private law.
[116] Article 12 Law 964 of 2005
[117] Article 33 Deceval Rule Book.
[118] Article 17 Deceval Rule Book

in Colombia the most suitable option is the closed ledger. Under this possibility, CSDs will still be performing as an important agent in the whole system, but with some changes on its duties. For instance, the CSD could design a ledger in which it will be only possible to participate in it, if there has been a previous authorisation to have access to the network. Those entities or inventors that the depository, or the regulator, consider appropriate to have authorisation to interact within the DLT will have their own copy of the ledger, rather than in a centralised party (CSD). Although the centralised data base will not be held by the CSD, this entity will become the custodian of the private keys that give access to the ledger, and also it will act as the controller of the network. With this approach, the custody of the securities will be delegated to each of the participants, but the control of how the network must operate and how the transactions are validated will be remain under the CSD's functions.

The design of an open ledger for clearing and settlement of securities could be highly risky if there are not appropriate controls and supervision from regulators. As we mentioned, in an open ledger the access is free to any person or entity without any limitation. In this scenario, this will open the door to many private companies, outside the scope of supervision and financial regulation, interested in designing a DLT network for the purpose of trade, clear and settle tokenised securities. In many cases, the access of any third party to this network will be unrestricted, without considering any control from supervisors, requirements or previous conditions. This may increase the systemic risk, counterparty risk, and liquidity risk, threatening the financial market´s stability. In this case, the regulator could prevent any hazardous activity by recognising other entities, besides those currently authorised by law, allowing them to perform clearing and settlement. This would only work if there is a requirement to fulfil conditions for authorisation to operate, in order to preserve the soundness and resilience of those entities, as well as submitting them to prudential supervision.

For this purpose, it is necessary to points out Philipp Peach's categorisation of "crypto-securities".

Secondly, once the DLT is accepted as an alternative to update the current post-trading infrastructure, regulators must address how to give legal recognition of ownership to those assets (securities tokens) recorded in the ledger. This issue will have to be addressed by aligning the current book entry system managed by the CSD with the DLT´s ownership recording system. As explained before, DLT is a system designed with the concept of decentralisation, under which each of the computers that are part of the network have their own trusted copy of the ledger. This means that a centralised party will not be necessary to validate and record each of the transactions that may affect the property of securities. In order to accomplish a valid transfer of ownership, the technology has its own mathematical algorithms under it is possible to design the manner in which each computer will communicate with other, how they will verify each transaction, and how they will have access to the network[119]. This mechanism of protocols, consensus and encryption allows the transferring of assets on a decentralised basis, where each party has its own trust copy of the ledger.

As we can see, the idea of a book entry system, understood as a centralised recording system of ownership managed by the CSD, may change since this entity will not be required to keep the registration of ownership. In this sense, regulation has to develop a new concept of ownership entitlement that will be valid and enforceable towards third parties. The ownership recording system within the DLT shall be legally accepted in harmonisation with the article 648 of the Commercial Code. On the other hand, it is also important that regulators have an overview of how other jurisdictions are dealing with this matter, to attempt a harmonisation in the operation of different networks around the world, as well as to avoid any conflict of laws at the moment of any cross border transaction.

Thirdly, to fully understand the previous challenges, it is also necessary to consider another aspect that is closely related to them. Previously, it was suggested that one of the big challenges that national regulators may face with the use of DLT is how to give legal recognition as securities to tokens. For this purpose, it is necessary to points out Philipp Peach's categorisation of "crypto-securities"[120].

First, the "Native Crypto Securities". These crypto-assets are those that the issuer has issued from the beginning of their existence as crypto-securities.

[119] EPaul Vigna, Michael J. Casey, "The Age of Cryptocurrency" (First Edition, St. Martins´s Press 2016)
[120] Philipp Peach (n 70)

Second, “Trans-Crypto Securities”. In this case, the issuer is moving securities that previously it has issued and are negotiated in the secondary market to the DLT environment. Thus, securities will have to mutate, and circulate as crypto-securities.

Third, “Intermediated Crypto-Securities”. There is a possibility that intermediaries, such as banks and brokers, will create an interface to settle transactions between themselves using a DLT infrastructure. To do this, those intermediaries are required to represent those securities as crypto-securities within the interface.

Hence it is necessary that policy makers and regulators will give the same treatment and recognition to crypto-securities as valid “securities” in coordination with the existing legal framework, which at the same time gives recognition of legal and enforceable property over those assets.[121] Particularly in the case of Colombia, this crypto-securities’ recognition is one of the main pillars, if the use of DLT or Blockchain technology to clear and settle securities is going to be considered, since market activities such as clearing and settlement rely on the condition that the assets to clear and settle must be recognised as securities.[122]

The law 964 of 2005 clearly expresses that securities are those assets that are characterised for being negotiable, for being part of an issuance process, and for having the purpose to raise money from general investors.[123] Although the law has a list of assets that are recognized per se as securities, the government has the possibility to give this classification to any other asset, not included in that list, if it fulfils the characteristics mentioned before.[124] This recognition is highly important, since the regulation is restricting clearing and settlement systems to those that have the purpose to clear and settle securities. It must be noted that if those crypto-securities

[121] Rosa María Lastra and Jason Grant Allen, ´Virtual Currencies in the Eurosystem: Challenges Ahead´ (European Parliament 2018) <http://www.europarl.europa.eu/committees/en/econ/monetary-dialogue.html> accessed 12 of June 2018.
[122] Article 9 Law 964 of 2005.
[123] Article 2 Law 964 of 2005.
[124] Article 4 (b) Law 964 of 2005.

are not recognized as securities, they will be outside of the law's scope, threatening investor protection, market resilience and market stability. For those reasons, it is fundamental that domestic regulation recognises crypto-securities as securities.

For instance, in some jurisdictions this work has been done with good results. Switzerland´s classification of tokens or crypto-securities could be used as a guide to how regulators should consider them. The first approach by the Financial Market Supervisory Authority (FINMA) considers the economical functions that underline the token creation. This is to define if it could be understood as securities, only if the token has been created to represent a debt or a claim against the issuer, since they could be treated similarly as shares, bonds or derivatives.[125] Along the same lines, La Comisión Nacional del Mercado de Valores in Spain and the Spanish Central Bank have categorized the token in two main types: 1) "Security Token", which represents a share in the revenues of a business and may be traded in a particular market; 2) "Utility Tokens", which are used as a mechanism to exchange for services or products.[126] Colombian regulators could consider recognizing tokens as securities, if those crypto assets are intended to be offered to general investors through issuance mechanisms such as IPOs, with the aim of being traded in the capital market. Regulating them and the use of DLT in clearing and settlement of securities could bring trust to the use of this technology enhancing liquidity problems because investors will be able to trade in a regulated market, reducing barriers at the moment of exchanging their investment to cash. [127]

Finally, it is important to mention how this new technology environment should interact with the finality principle in the post-trading. In Colombia, this principle is applied in those securities transactions where the order for transferring securities and cash has been received and accepted by the clearing and settlement system.[128] This means, according to domestic law, that the accepted transfer orders are considered unconditional, irrevocable and cannot be affected by any insolvency circumstance, guaranteeing by this a final settlement.[129] This is used to ensure that the Colombian system operates on a safe and efficient basis for the securities delivery and fund payments. [130] Therefore, it is necessary that in the DLT regulations will be established once the consensus is achieved, this means once the validation of the transaction has taken place, the finality principle must take place. This will depend on how the protocol and the regulations are designed accordingly. Also, by regulatory requirement, this protocol and consensus should be approved by the Financial Superintendence before entering into force.

[125] AFINMA, ´Guidelines For Enquiries Regarding The Regulatory Framework For Initial Coin Offering (ICOs)´ (16 February 2018) <https://www.finma.ch/en/news/2018/02/20180216-mm-ico-wegleitung/> accessed 22 July 2018.
[126] Banco de España and Comisión Nacional del Mercado de Valores, ´Joint press statement by CNMV and Banco de España on "cryptocurrencies" and "initial coin offerings" (ICOs)´ (February 2018) <http://www.cnmv.es/portal/verDoc.axd?t={6f310cc7-6b39-4405-a8f7-70d2b1e682d1}> accessed 22 July 2018.
[127] Awery Van Zwieten Final, ´The Shadow Payment System´ (2016) 43 Journal of Corporation Law 101.
[128] Article 10 Law 964 of 2005.
[129] Nydia Remolina Leon, ´Liquidación y Compensación anticipada de Derivados OTC en Caso de Insolvencia Análisis Económico y Jurídico del Artículo 74 de la Ley 1328 de 2009´ (2009) Revista de Maestría de Derecho Económico.
[130] German Abella Abondano (n 98).

04 | Distributed Ledger in MILA

The use of technology within capital markets has permitted interaction among different markets so that today it is possible to think about a globalized securities market. Technology advances have allowed securities or any other type of assets to be purchased in different jurisdictions, without it being necessary to travel thousands of miles. One example of this incredible development is the Mercado Integrado Latinoamericano (MILA), which was put in place in 2009 through an agreement signed among the stock exchanges and CSDs from Chile, Peru and Colombia with the idea of creating an integrated regional market.[131] After a couple of years, the integrated market entered into operation in 2011, and later, in 2014, the Mexican stock exchange and its CSD became part of the joint market.[132]

This integration, among others, is evidence of how important a common regulatory language is in securities markets. Thus, in order to accomplish this, and to create more liquidity, it was necessary to agree on how a different technology system would interact; the result was the creation of a single platform with a dedicated communication channel located in each country.[133] However, this process was not an easy task since some difficulties arose such as: 1) differences in the accounting registration system; 2) Institutional investors, in some of the jurisdictions, had technological incompatibilities with the MILA infrastructure, which brought difficulties for the interaction with the market; and 3) differences in tax regulation treatment.[134] For this reason, the implementation process took time to settle in, until each jurisdiction accommodated their infrastructure in order to create compatibility within the legal systems and technological requirements.

The whole implementation was divided into two main phases:[135] Phase one was the adoption of a dedicated communication channel, in which the broker located in each jurisdiction could route the orders to the foreign broker who would be responsible for including the selling or buying order in its local stock exchange. For example, a Colombian investor wanting to buy shares in the Peruvian market has to give the order to his local broker, who will channel the requirement to a broker located in Peru through a dedicated communication channel. Afterwards, the local broker, the Peruvian broker, will include the order in his local stock exchange, in this case La Bolsa de Valores de Lima. In the Second Phase of the integration, the ideal scenario is to facilitate the link between the foreign broker and the local exchange by allowing brokers to have direct access to each of the local markets without intermediation.

To illustrate the process of how MILA operates, it is important to understand that in every transaction it is necessary that the investor's broker has a correspondent broker in the market where the securities are listed, and it is this latter who will process the order in the local stock exchange.[136] Therefore, the custody of those securities are held by a local CSD in the jurisdiction where the securities have been issued. At the same time, this CSD has an omnibus account in the foreign CSD. Then, once a securities transaction has taken place - foreign investor buying or selling securities - the local CSD debits or credits the foreign CSD's omnibus account, who then replicates the transaction in the foreign investor's account

[131] MILA, ´Quienes Somos: Historia´ <http://mercadomila.com/quienes-somos/resena-historica/> accessed 22 July 2018.
[132] Ibid.
[133] Jose Fernando Romero, ´Mercado Integrado Mila, Motor de la Alianza del Pacifico´ (2015) Universidad Peruana de Ciencias Aplicadas.
[134] Beatriz Yepes-Rios, et al. ´The Integration of Stock Exchanges: The Case of Latin America Integrated Market (MILA) and Its Impact on Ownership and Internationalization Study in Colombia Brokerage Firms´ (2015) 20 Journal of Economic, Finance and Administrative Science 84.
[135] Mila, ´Documento General del Producto Mercado Integrado Latinoamericano (MILA)´ Bolsa de Valores de Colombia <http://www.bvc.com.co/recursos/Files/Mercados/Renta_Variable/Guia_MILA.pdf>
[136] Ibid

by debiting or crediting it.[137] Consequently, settlement of each transaction in the MILA market is done by following the domestic laws applied to the local CSD; jurisdiction where the securities were registered.[138] As we can see, this structure requires an appropriate interconnection system and harmonization in the legal framework that facilitate the communication protocols among intermediaries, as well as clear rules that govern the procedures which establish each party's responsibilities in the process. DLT may create a new process to redefine how the MILA market shall be operated with new technology infrastructure that perhaps will change how the system is built today. For instance, this technology will facilitate the direct interaction of the brokers in the MILA market. But the role of CSD will be reduced since they will just become the managers of the ledger, monitoring that each node (Brokers, CSDs, and Custodians) operates according to the protocols and rules established in the ledger.

137 MILA, "Cómo se Custodian Mis Valores MILA?" <http://mercadomila.com/wp-content/uploads/2018/04/MILA_Custodia_de_Valores.pdf> accessed 24 July 2018.

138 MILA, "Liquidación de Valores en MILA" <http://mercadomila.com/wp-content/uploads/2018/04/MILA_Liquidacion_de_Valores.pdf> accessed 24 July 2018.

The MILA market has been strongly supported in each of the countries who are members of the integration

The MILA market has been strongly supported in each of the countries who are members of the integration, since it has been an opportunity to bring a number of benefits to their economies. The integrated market is a big step towards increasing the appetite for investment in the region, making the financial market bigger and more competitive.[139] It can be pointed out that this process carries positive things such as:[140] i) the development of the stock markets, ii) Reduction in the operational cost; iii) incentive to more cross-border listing; iv) facilitates portfolio diversification; and creates incentives for legal and regulatory harmonization. However, MILA has also created some challenges that even today are important to consider. Those are:[141] i) domestic regulatory framework should aim to guarantee investors the possibility of exercise¡ing their shareholders rights toward the foreign issuer; ii) self-regulation has to be compatible with the new operation of the system and international laws; and iii) the presence of uncertainty in the securities holding, and securities' ownership.

´Financial Integration in Latin America´ (March 2016) https://www.imf.org/external/np/pp/eng/2016/030416.pdf accessed 23 March of 2018.

[140] Daniel Jacob Leraul, ´Trading With Neighbors: Regional Stock Exchanges Integration – The Mercado Intergrado Latinoamericano´ (2016) 17 (1) Latin American Business Review 49.

[141] Mauricio Baquero-Herrera, ´Legal Certainty and Financial Market Integration: The MILA Case´ (2013) 19 Law and Business Review 487.

[142] DCV, 'CSD Working Group Sobre DLT Suscribe Memorando de Entendimient' (17 January 2018) <https://www.dcv.cl/es/centro-de-noticias/avances-de-noticias/articulos/3715-csd-working-group-sobre-dlt-subscribe-memorando-de-entendimiento.html> accessed 24 July 2018.
[143] Sergio Clavijo, ´Integración Bursátil en la Región Andina´ (2010) 46 Enfoque Mercado de Capitales <https://www.deceval.com.co/portal/page/portal/Home/Gestion_Corporativa/informe_de_investigacion/2010/Enfoque46-10.pdf > accessed 14 July 2018.
[144] Chad P. Bown, et al. Better Neighbors Toward a Renewal of Economic Integration in Latin America (World Bank Publications, The World Bank Group 2017) <https://openknowledge.worldbank.org/handle/10986/25736> accessed 25 July 2018.

DLT could be an alternative to overcome those challenges. At the beginning of 2018, the Chilean CSD signed a Memorandum of Understanding with other CSDs and intermediaries to work jointly in the implementation of this technology in post-trading services.[142] This evidence in the application of new technologies as DLT within the regional securities markets could be a good moment to contemplate harmonizing the legal framework applied to cross-border transactions performed in MILA. To do this, clearing and settlement systems must have clear and definitive procedures among foreign and local participants that guarantee a simultaneous settlement based on finality and certainty.[143] This technology is still new and has many ambiguities, but markets cannot be blind to new developments, so they need to see how they can use it with the aim of creating a deeper and more liquid capital market.

As we have seen, the MILA market requires the interaction of different intermediaries within each layer of the process. DLT could help to simplify the architecture towards accomplishing the second phase of the integration. Nevertheless, regulators, market participants, and intermediaries will need to collaborate between each other in order to identify the regulatory and procedures mismatches to create a common understanding. MILA, as well as in Colombia, needs a harmonization of each national legal framework and procedures with the new technology requirements. In this case, thanks to the integration of the market, the task could be more complex since there is more than one jurisdiction involved. However, the current agreement achieved by the Colombia, Chilean, Peruvian and Mexican markets during the integration process may be, on the other hand, a good approach to answer some of DLT's challenges. This is: i) defining ownership and safe keeping requirement; ii) establishing a common acceptance tokens concept as securities. This will need amendment processes in each national law; iii) Identifying and clarifying the settlement finality; and iv) DLT has to be integrated in parallel with the current system. Both systems must be compatible, based on good industrial practice and international standards.

In a recent report issued by the World Bank, it was suggested that some of the issues that have created difficulties in achieving a fully integrated market in Latin America has been the lack of common currency, and differences in legal origins.[144] Although in the same report the constant increase in portfolio investment within the region has been highlighted, it is concluded as well that Latin American countries must work in aspects such as increasing national productivity and investing in more innovation. Thus, bearing these recommendations in mind, it is possible to consider MILA as a good tool for capital market integration that could improve even more if new technologies are used to channel more capital flows, creating more liquid and diversified regional capital market. DLT could help to harmonize MILA infrastructure, reducing cost and attracting more participation of foreign investors not only within the region, but also from other international markets. Since this technology could reduce costs and simplifies processes, it could be the appropriate scenario in which regulators and government reach an agreement in common regulatory standards, by guaranteeing better regional production levels, investment, and financial inclusion.

Conclusion

To conclude, this document has intended to analyse the DLT´s legal implication in the post-trading of securities, mainly focused in the Colombian post-trading infrastructure, as well as in the MILA market. An extensive number of studies have been written by different authorities who have analysed the use of DLT or Blockchain in the clearing and settlement of securities in different legal systems. A common assumption in those studies suggests that DLT may bring a number of benefits to the current system, such as efficiencies in the post-trading architecture and cost reduction in the value chain. Thanks to the benefits that this technology may bring to capital markets, many jurisdictions and regulators are assessing the idea of applying DLT or Blockchain technology in their current clearing and settlement of securities system. Therefore, since in Colombia today there has still been no research to assess the use of this innovative tool, this research has intended to provide the first approach to this analysis in the Colombian context, as well as in the MILA market.

On the other hand, DLT supporters are arguing that this shared network is the first step towards rebuilding capital markets in which intermediation will not be necessary any more, threatening the existence of Central Securities Depositories, Central Counter Parties, and Brokers. However, it has been analysed that this idea is not completely accepted for the industry. It has been suggested that instead of disappearing, what is possibly going to happen is that today's perception on how we used to conceive them will change. Especially, regarding the role that they play in today's system.

In the case of the Colombian market infrastructure, it is important to conclude that intermediation is still necessary since it guarantees certainty and confidence for investors that the market is still regulated and supervised in order to be protected from systemic risk, and misappropriation of resources. Also, it has been highlighted that the adoption of DLT will bring some regulatory challenges for regulators with regards to redefining settlement finality principle, clarifying ownership and safe-keeping of securities, and the recognition of tokens as securities under the domestic legal framework. Also, it has been stated that DLT may bring benefits for the regional integration of capital markets (MILA), attracting more issuers and investors, improving capital allocation and investment environment in the region.

This document has analysed the legal implication of DLT in a market that is still increasing in size.

This document has analysed the legal implication of DLT in a market that is still increasing in size. New challenges will appear as this technology will be used in other markets. However, this will require new common standards that allow the interoperability of different DLT infrastructures in a cross-border transactions, bringing certainty and legal protection for investors.

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